

Multidimensional poverty indices: A critical assessment

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Motivation
Index properties
Other indices
Dominance

Outline

Motivation

Constructing
indices

Measuring
multidimensional
poverty

Poverty frontiers

Counting poor
people/poor
dimensions?

$H(\zeta)$ and $M(\zeta)$ with
2 dimensions

Index properties

Other indices

Dominance

Motivation

A few issues:

- Aggregate dimensions or individuals first?
- Aggregation of dimensions: sensitive to dimensional inequality?
- Aggregation of dimensions into an individual measure of well-being/deprivation?
- Identification: set multiple poverty lines/one poverty line for multidimensional wellbeing?
- Identification: focus on union/intersection/intermediate measures?
- Aggregation of individuals: sensitive to inequality across individuals?

Measuring multidimensional poverty

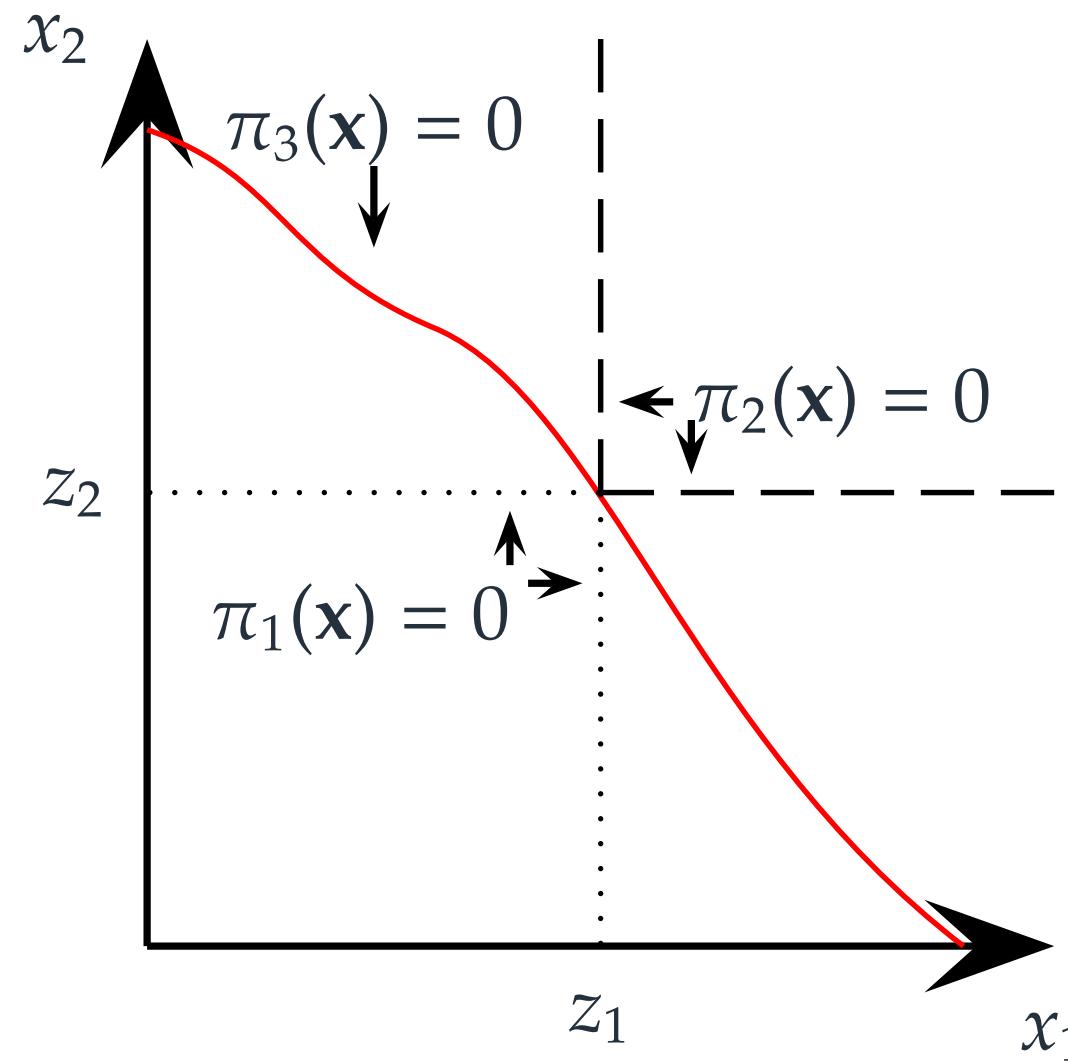
- Index of multidimensional poverty: $P(\mathbf{X})$
- \mathbf{X} is $N \times K$ matrix; \mathbf{x}_i is vector for i
- Distribution function: $F_{\mathbf{X}}(\mathbf{x})$
- Additive multidimensional poverty indices:

$$P(\mathbf{X}; \mathbf{z}) = \int \pi(\mathbf{x}; \mathbf{z}) dF_{\mathbf{X}}(\mathbf{x}), \quad (1)$$

$\pi(\mathbf{x}; \mathbf{z})$ is individual-level aggregation of deprivation.

- For discrete distributions:

$$P(\mathbf{X}; \mathbf{z}) = N^{-1} \sum_{i=1}^N \pi(\mathbf{x}_i; \mathbf{z}). \quad (2)$$



Counting poor people/poor dimensions?

- Let ζ be the number of dimensions in which someone needs to be deprived to be considered as multidimensionally poor.
- $H(\zeta)$ is traditional multidimensional headcount:

- Union:

$$H(1) = 1 - \int_{\mathbf{z}}^{\infty} dF_{\mathbf{X}}(\mathbf{x}). \quad (3)$$

- Intersection:

$$H(K) = \int_0^{\mathbf{z}} dF_{\mathbf{X}}(\mathbf{x}) = F_{\mathbf{X}}(\mathbf{z}). \quad (4)$$

Counting poor people/poor dimensions?

- Alkire-Foster index (AF, which depends on \mathbf{z} , ζ and α) given by:

$$\pi(\mathbf{x}_i; \mathbf{z}) = K^{-1} \sum_{k=1}^K g_{i,k}^\alpha I(d_i \geq \zeta) \quad (3)$$

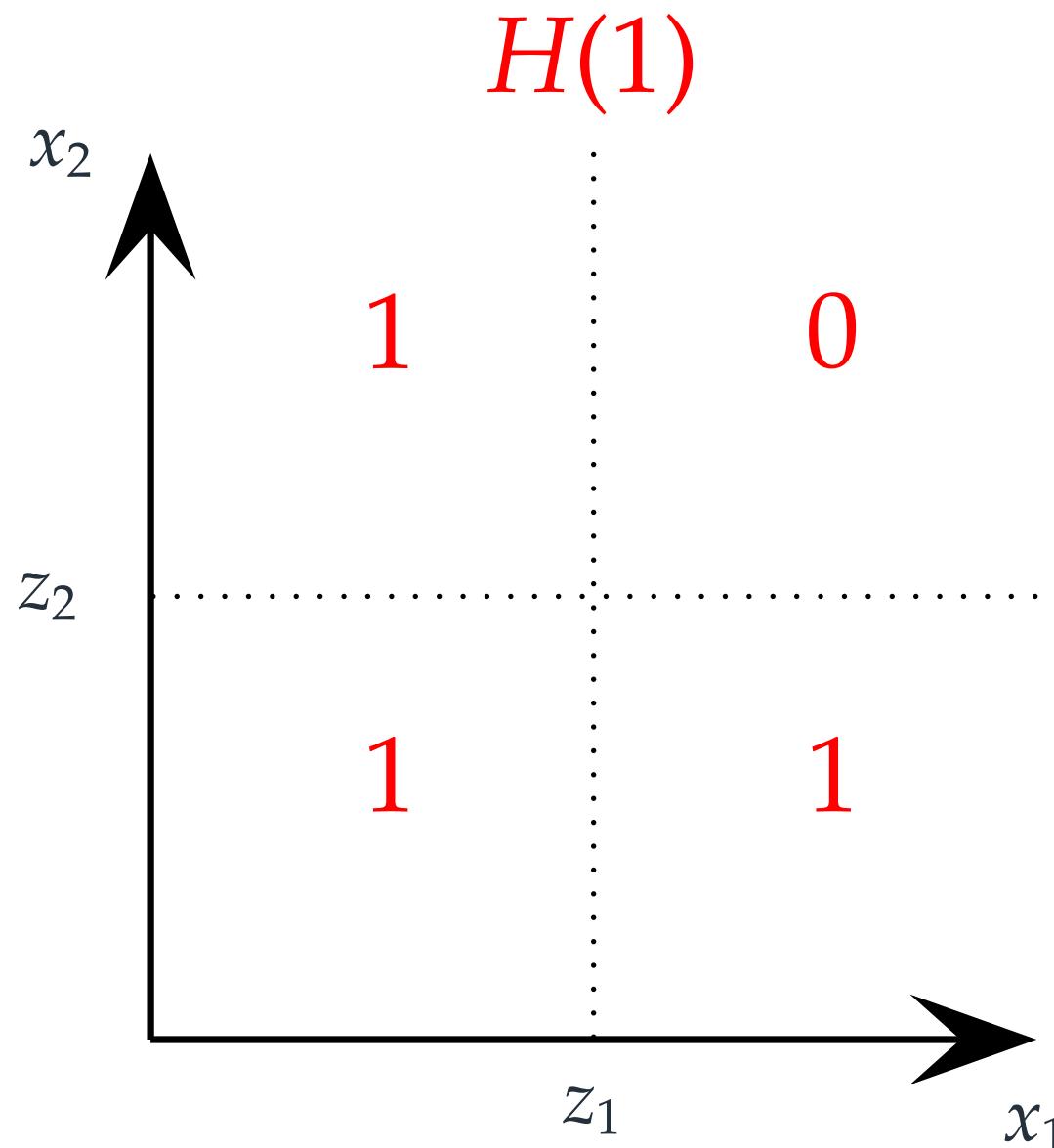
(supposing dimensions have equal weights) where: $g_{i,k}$ is poverty gap of i in k ; $d_i = \sum_k g_{i,k}^0$ denotes the number of dimensions in which individual i is deprived.

Counting poor people/poor dimensions?

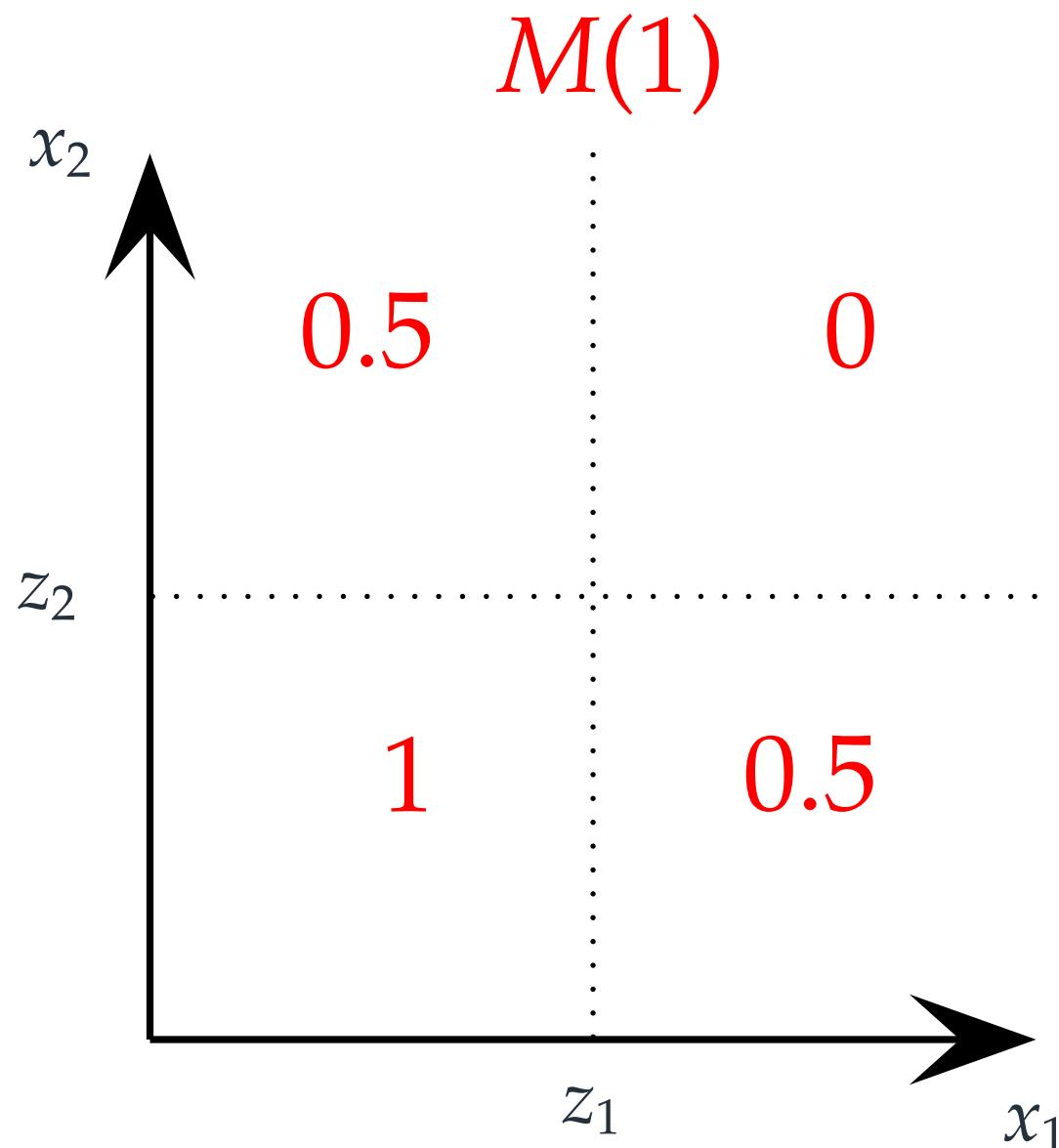
- UNDP's MPI index ($M(\zeta)$) is a special case of AF, obtained with $\alpha = 0$:

$$\pi(\mathbf{x}_i; \mathbf{z}) = \frac{d_i}{K} I(d_i \geq \zeta) \quad (3)$$

$H(\zeta)$ and $M(\zeta)$ with 2 dimensions

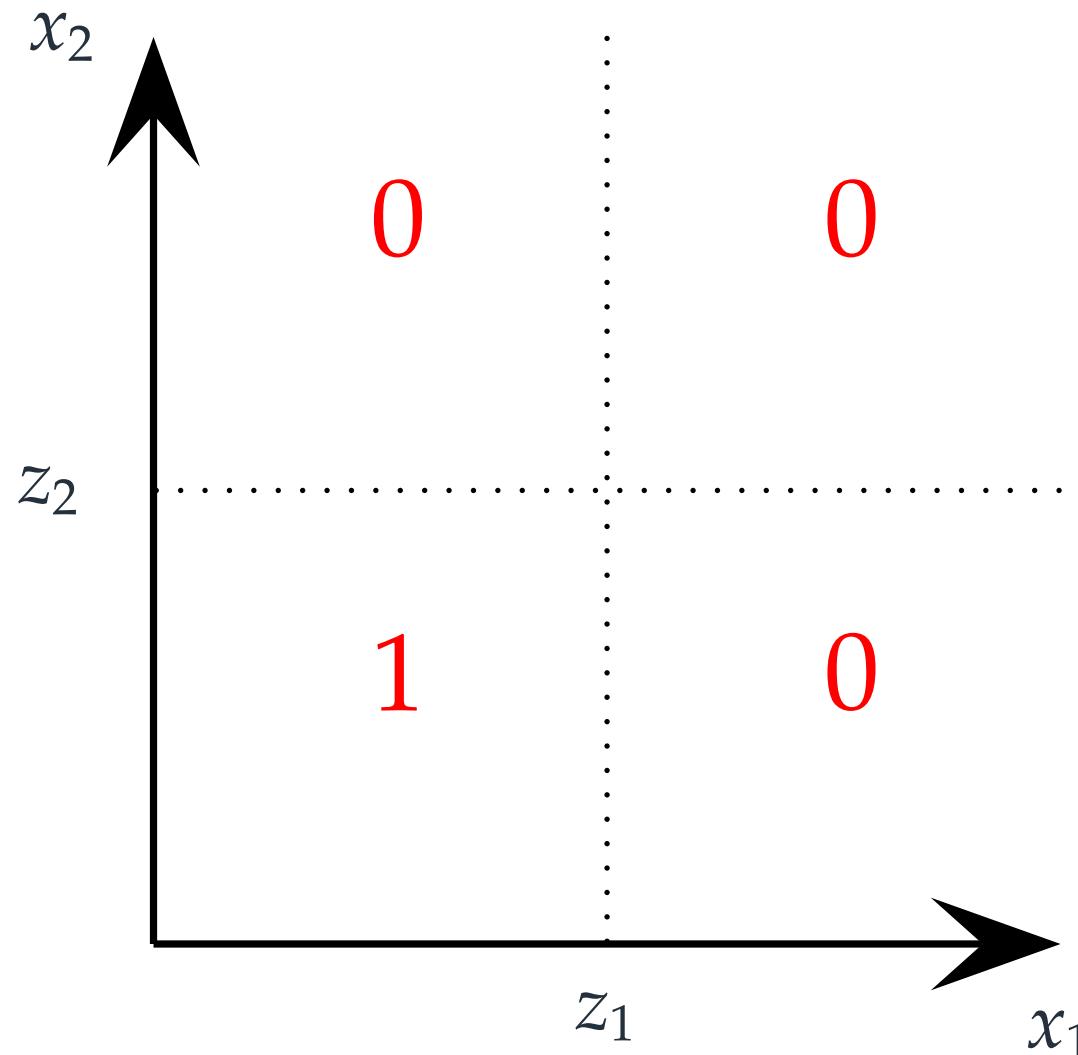


$H(\zeta)$ and $M(\zeta)$ with 2 dimensions



$H(\zeta)$ and $M(\zeta)$ with 2 dimensions

$$H(2) = M(2)$$



Outline

Motivation

Index properties

Most popular axioms

H -type and ζ -type discontinuities

Unidimensional equalization decreases poverty

Unidimensional equalization can also increase poverty?

Multidimensional equalization can increase poverty

Decrease in correlation

Equalization can increase/decrease MPI

3D discontinuities of $M(2)$

But increasing correlation can also increase $M(2)$

Multidimensional poverty
Other indices

Index properties

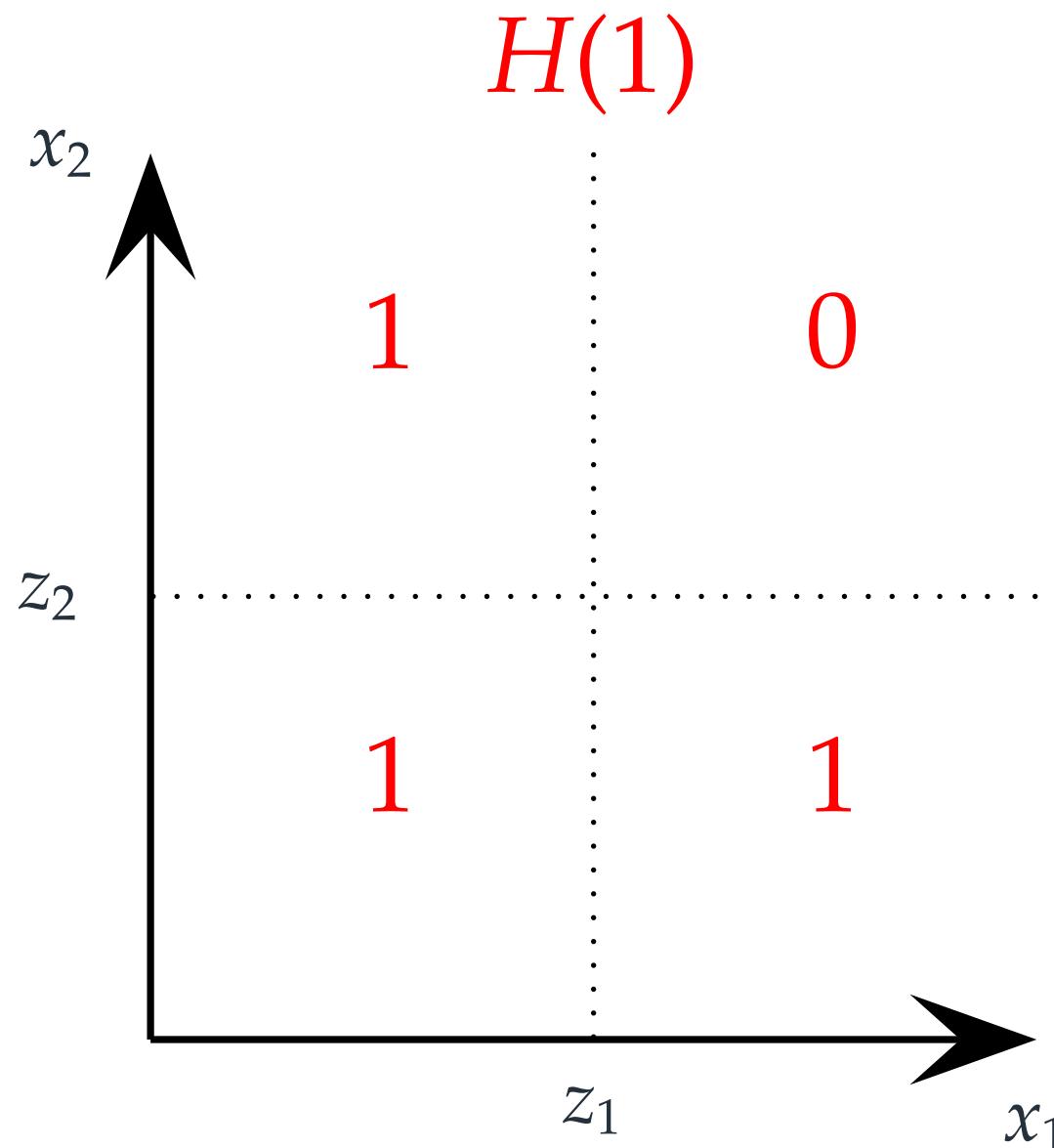
Most popular axioms

- Anonymity: permuting of X does not change poverty.
- Population: replicating matrix X several times leaves poverty unchanged.
- Scale invariance

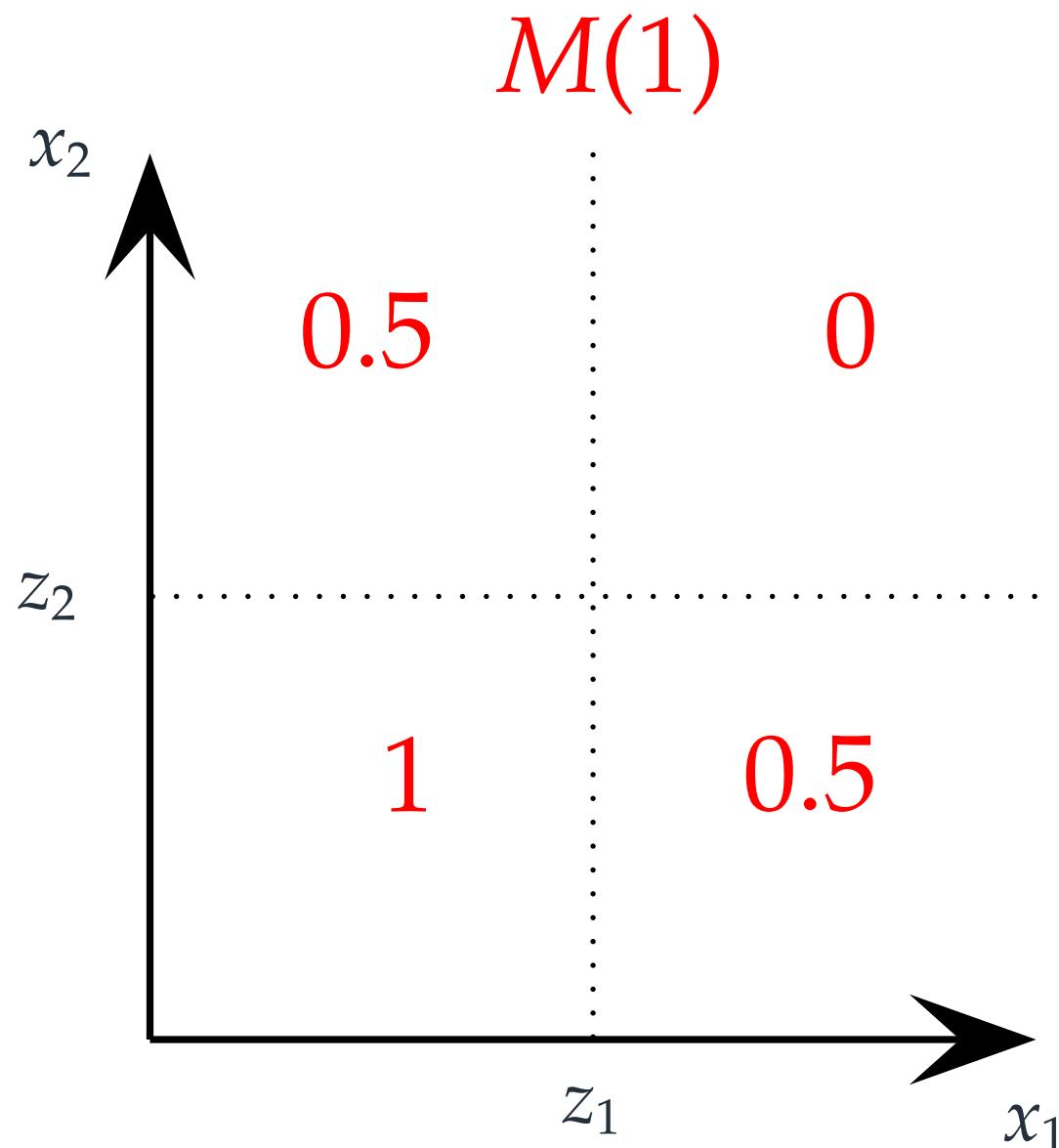
Most popular axioms

- Continuity: small measurement errors do not cause large changes in poverty.
- Monotonicity: poverty declines if a poor attribute of a poor individual increases.
- Unidimensional equalization should decrease poverty.
- Multidimensional equalization should decrease poverty.
- Correlation sensitivity: the greater the correlation between attributes, the greater is poverty.

H -type and ζ -type discontinuities

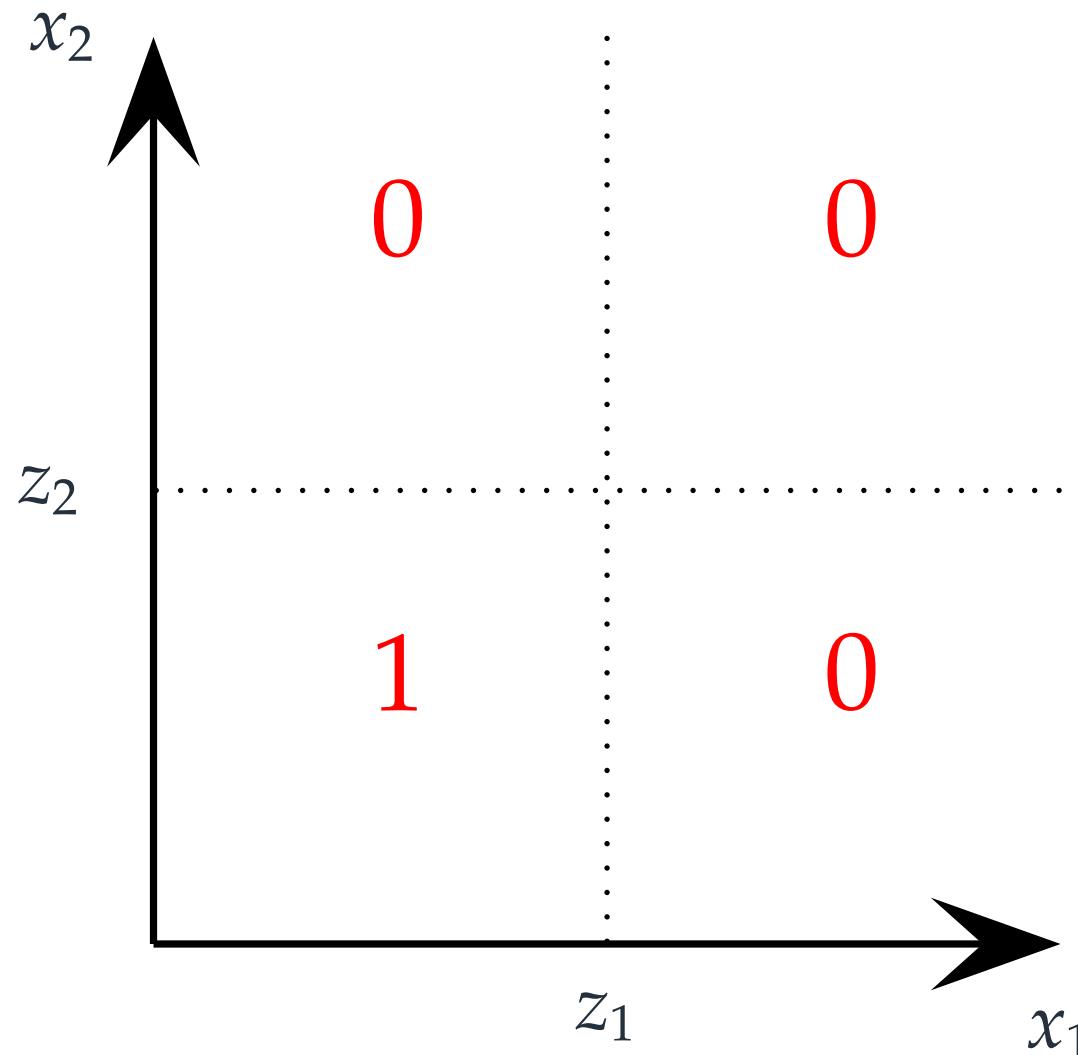


H -type and ζ -type discontinuities

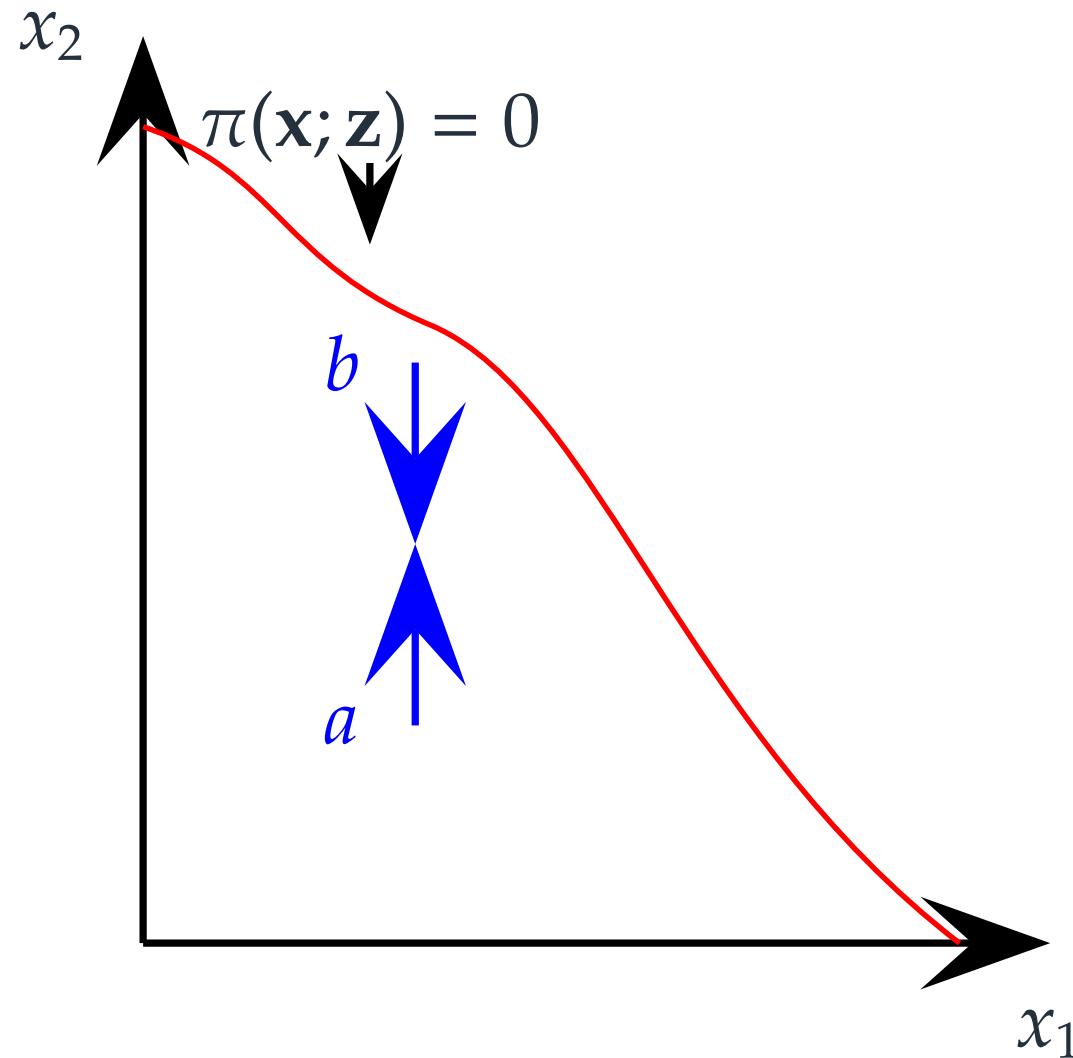


H -type and ζ -type discontinuities

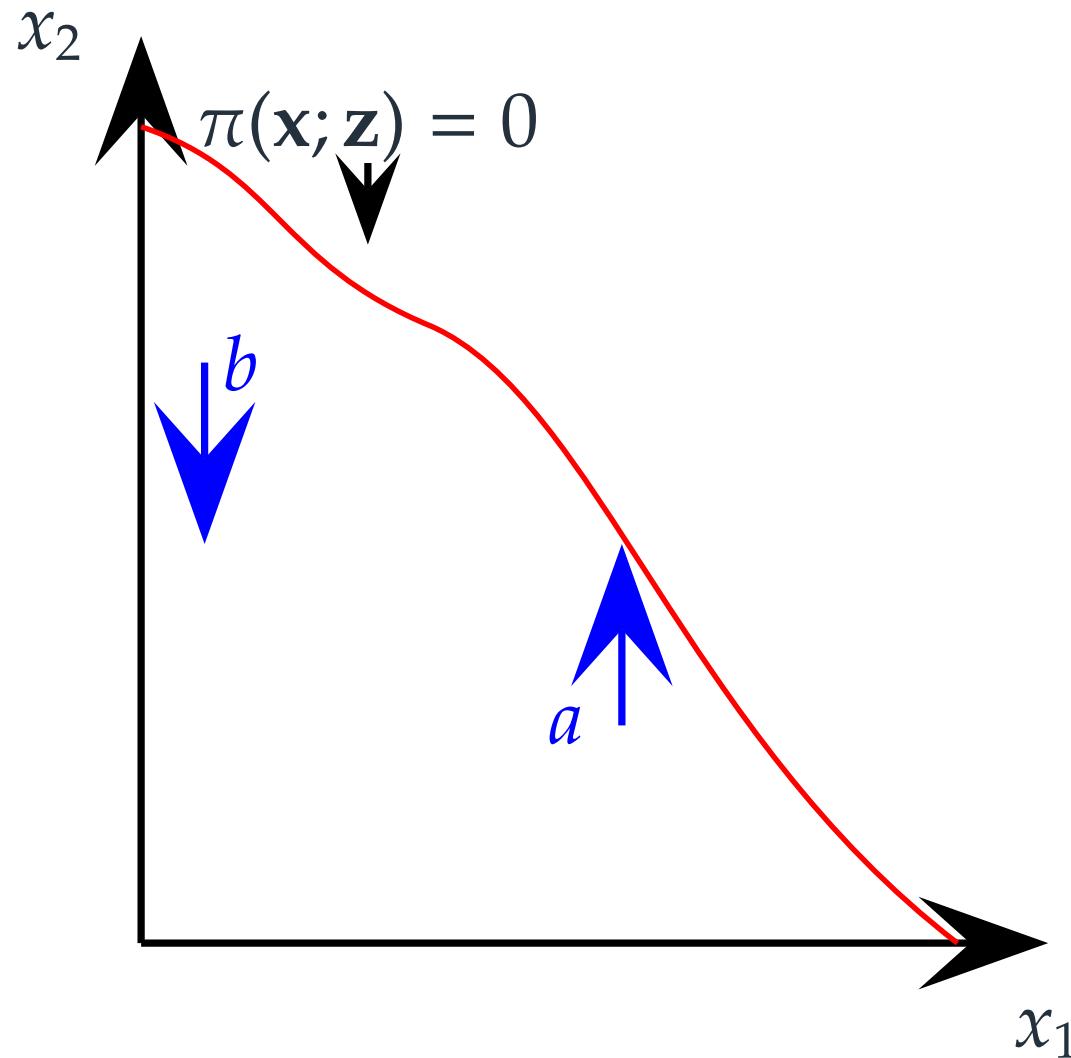
$$H(2) = M(2)$$



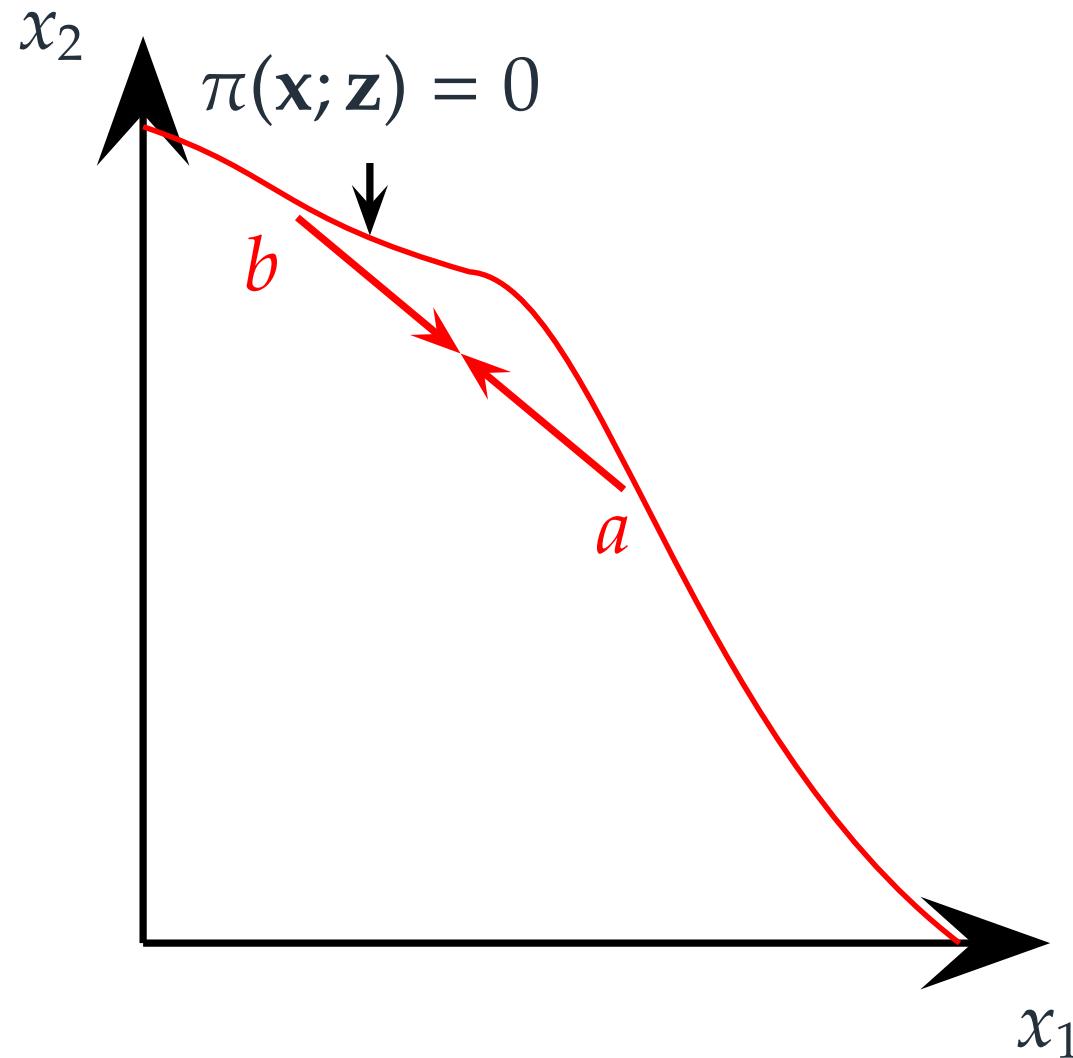
Unidimensional equalization decreases poverty



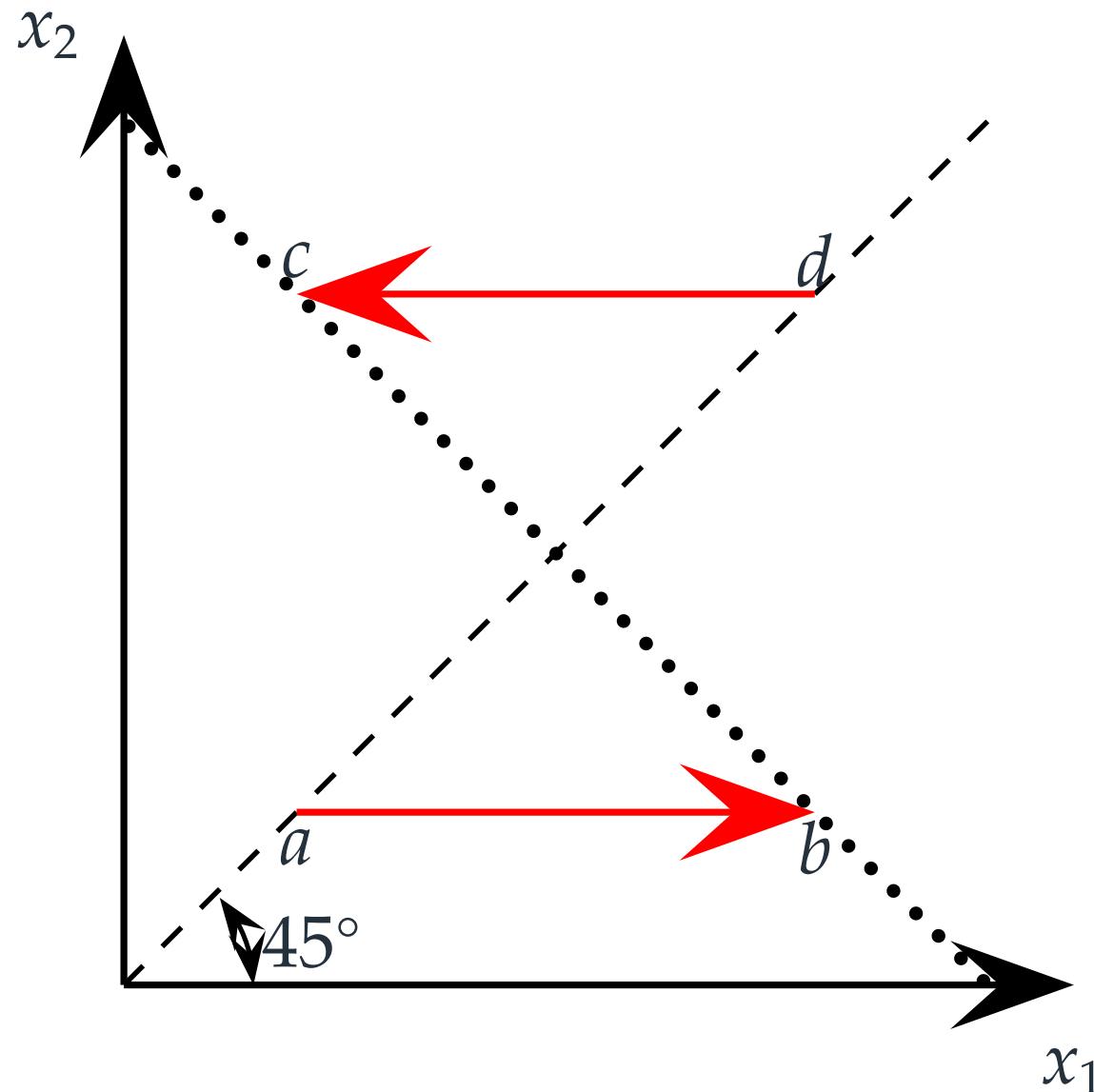
Unidimensional equalization can also increase poverty?



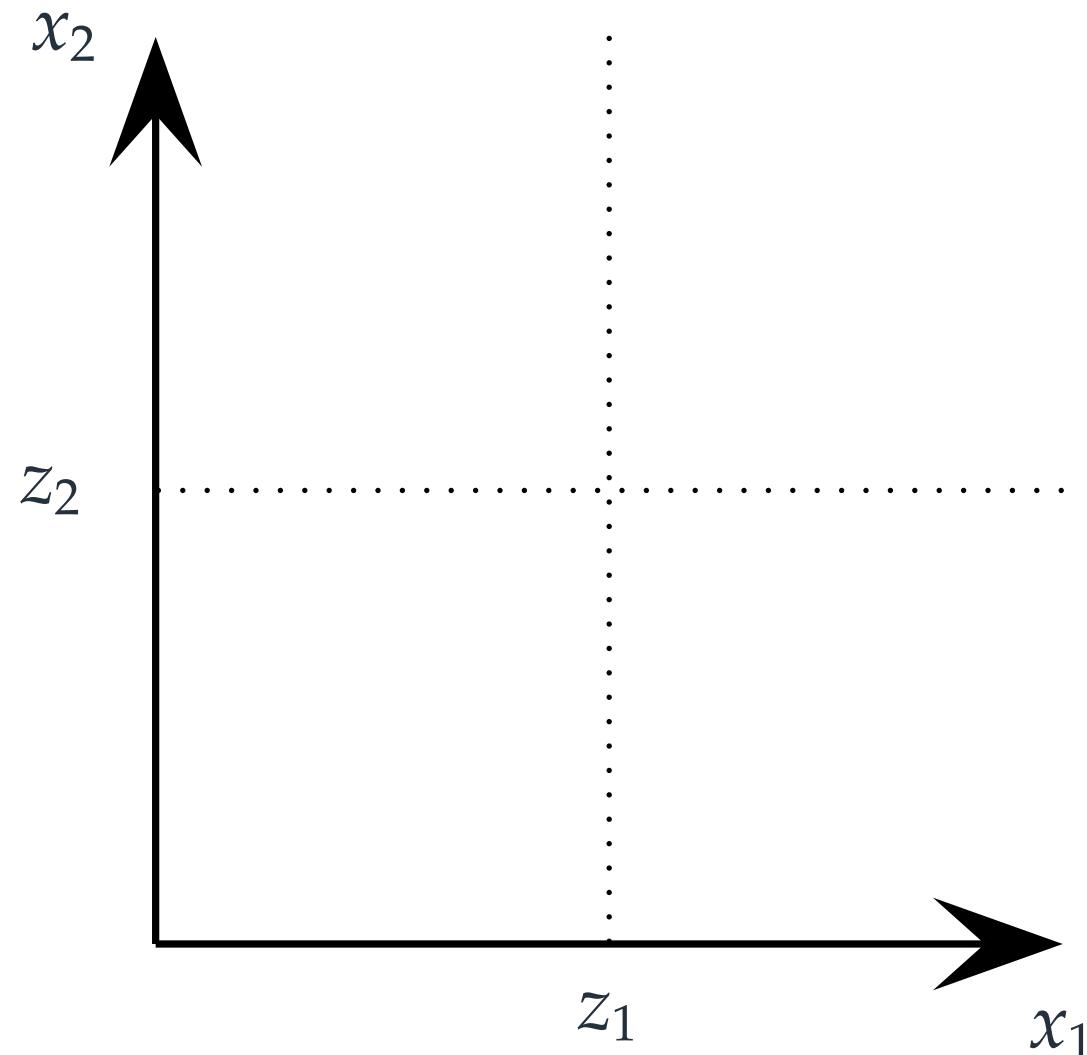
Multidimensional equalization can increase poverty



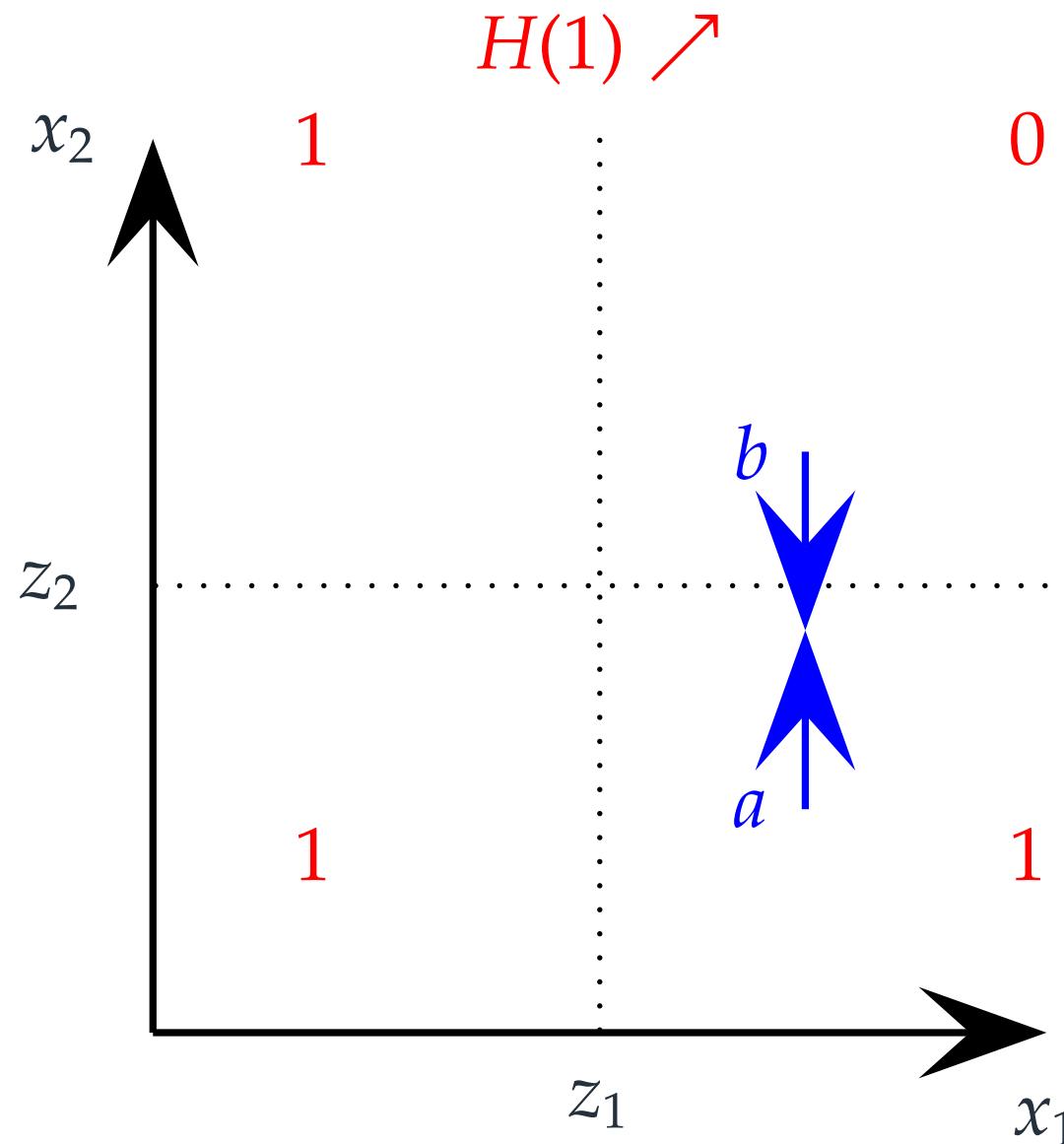
Decrease in correlation



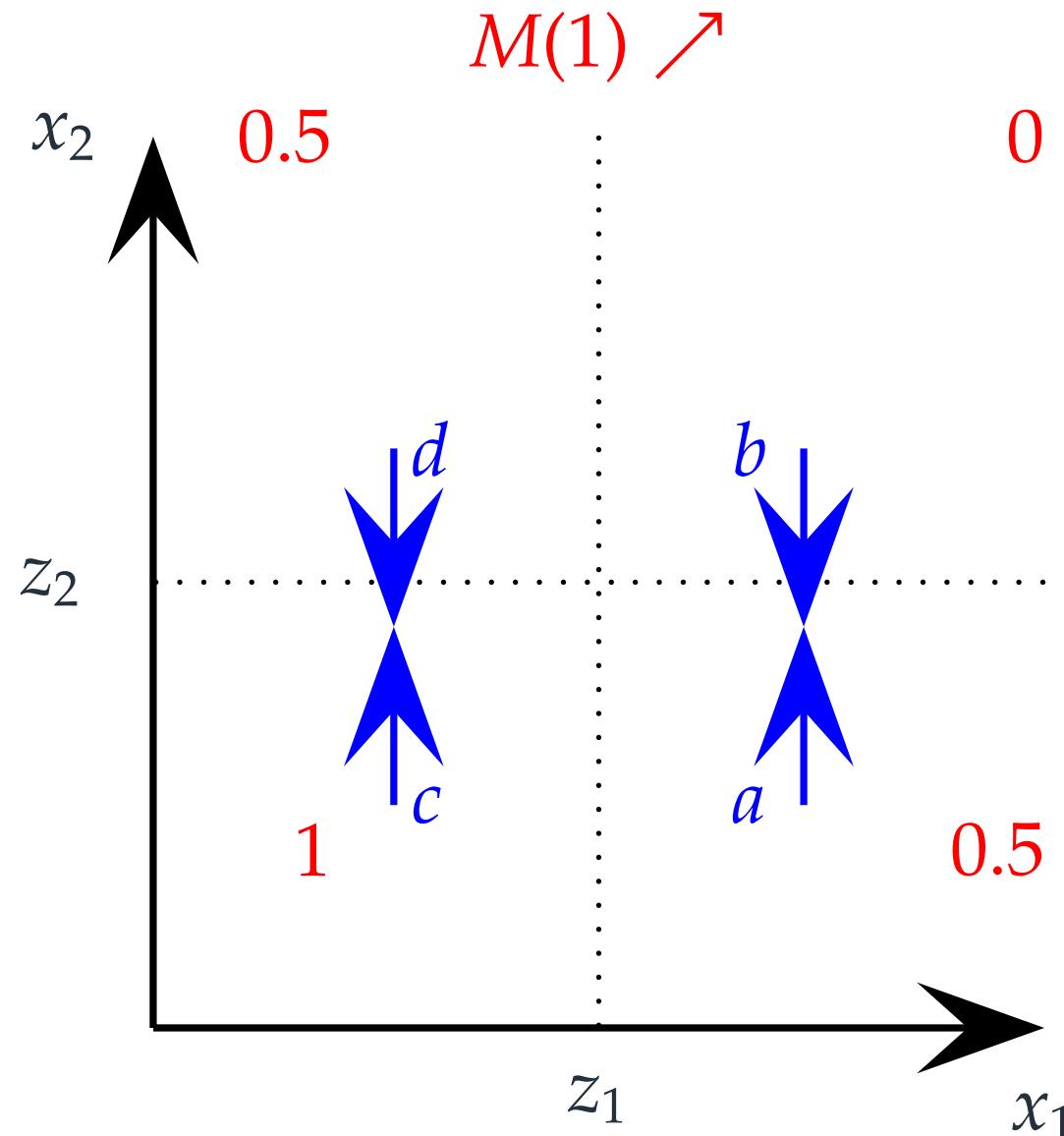
Equalization can increase/decrease MPI



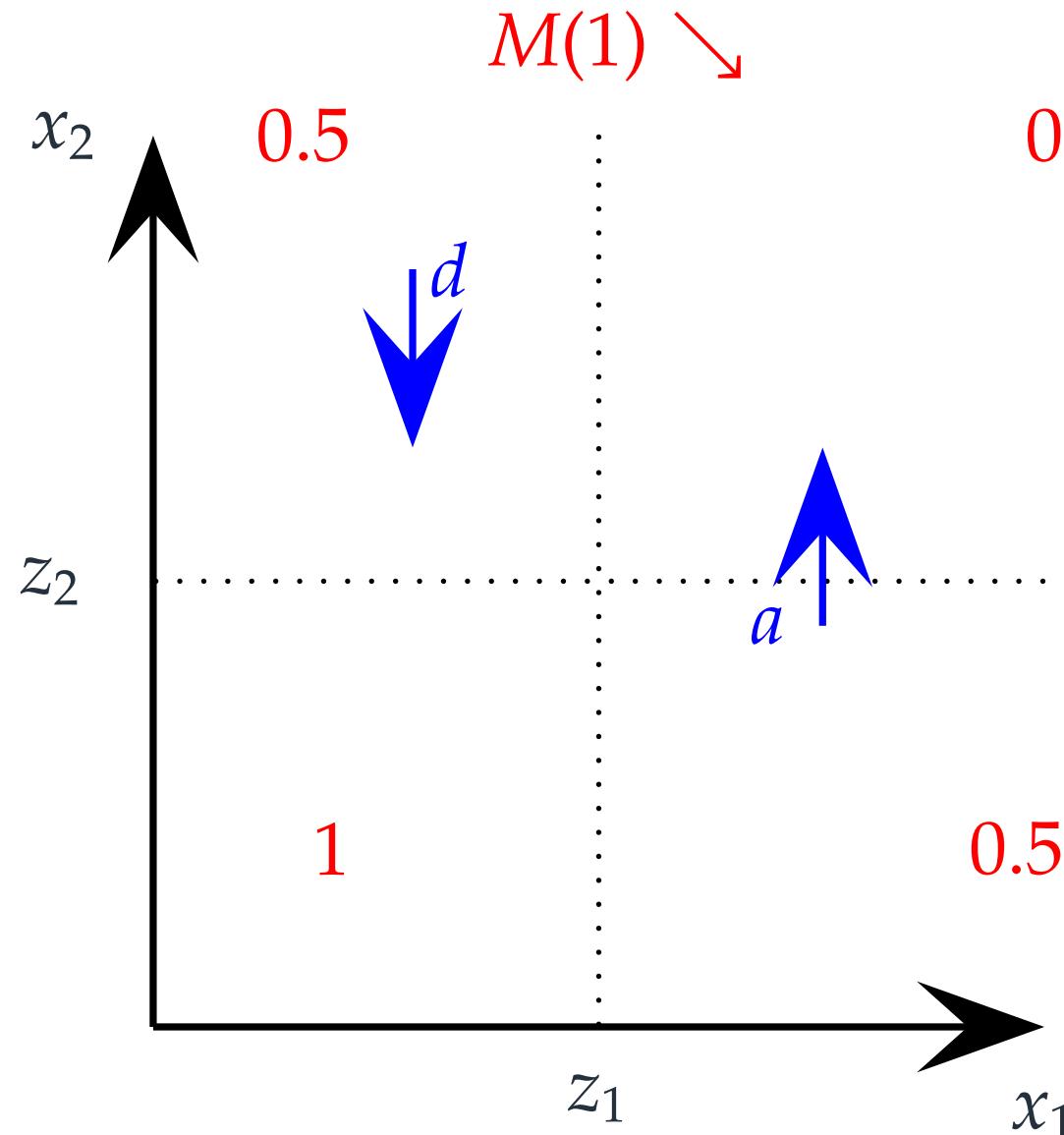
Equalization can increase/decrease MPI



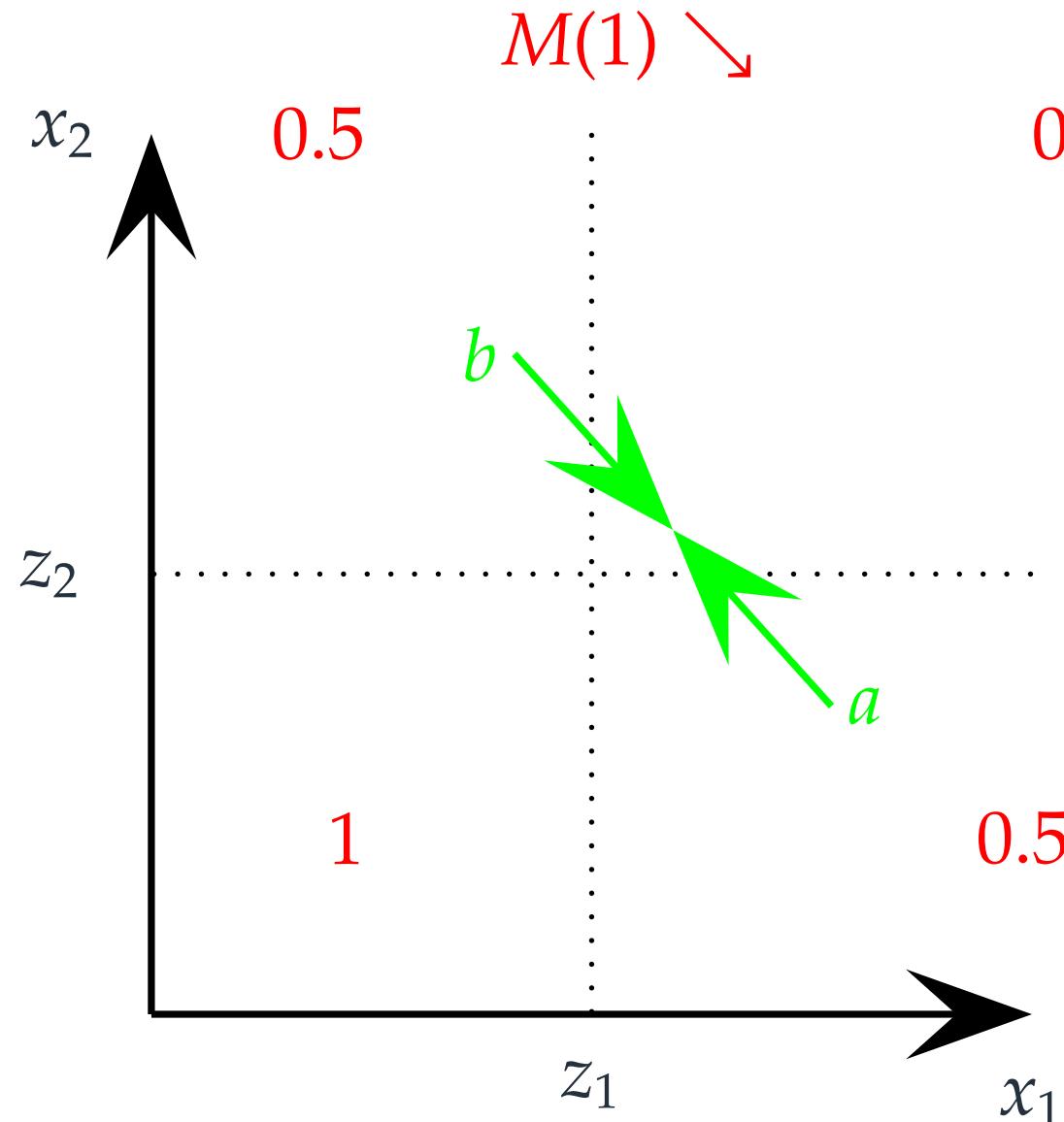
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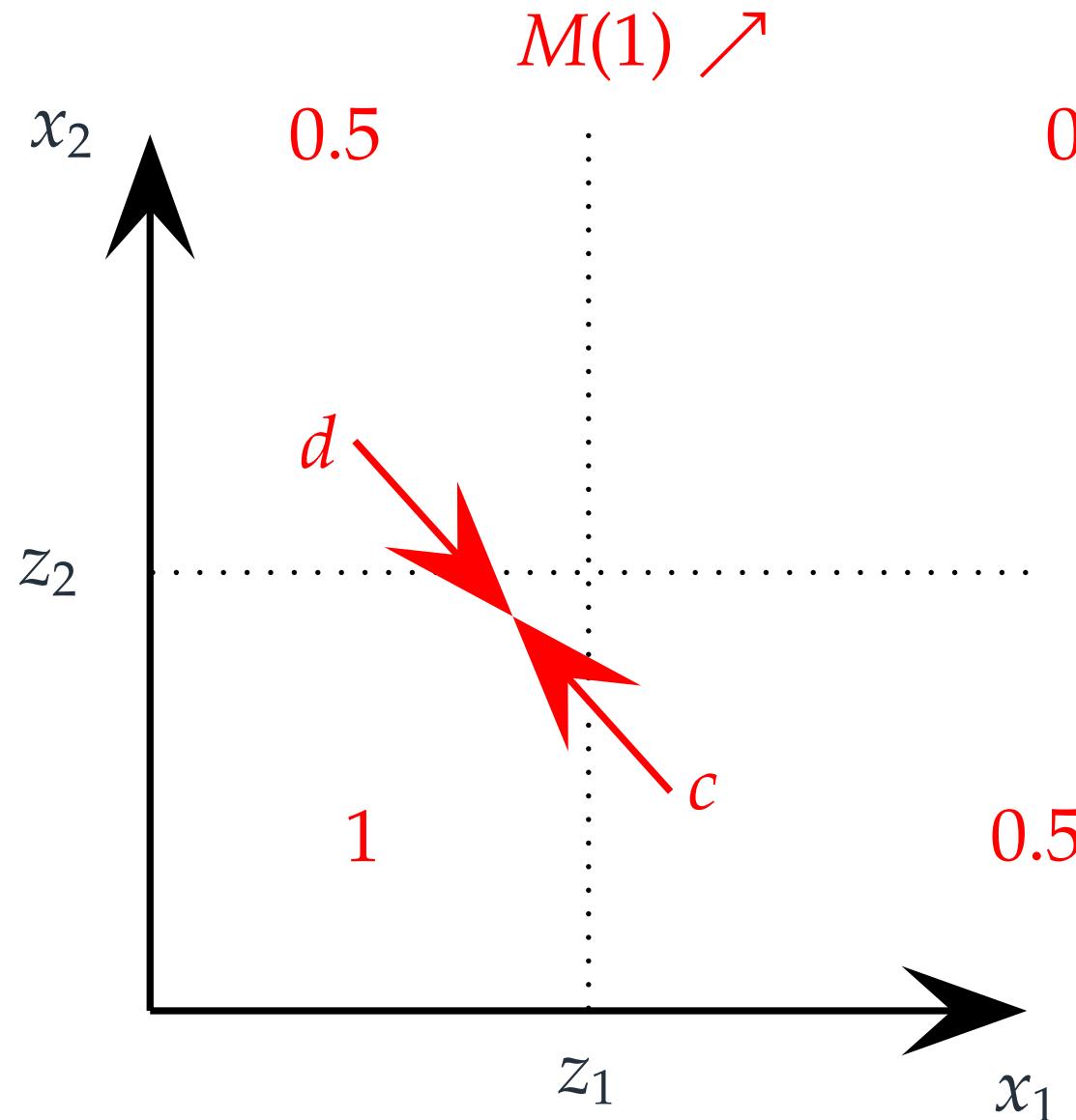
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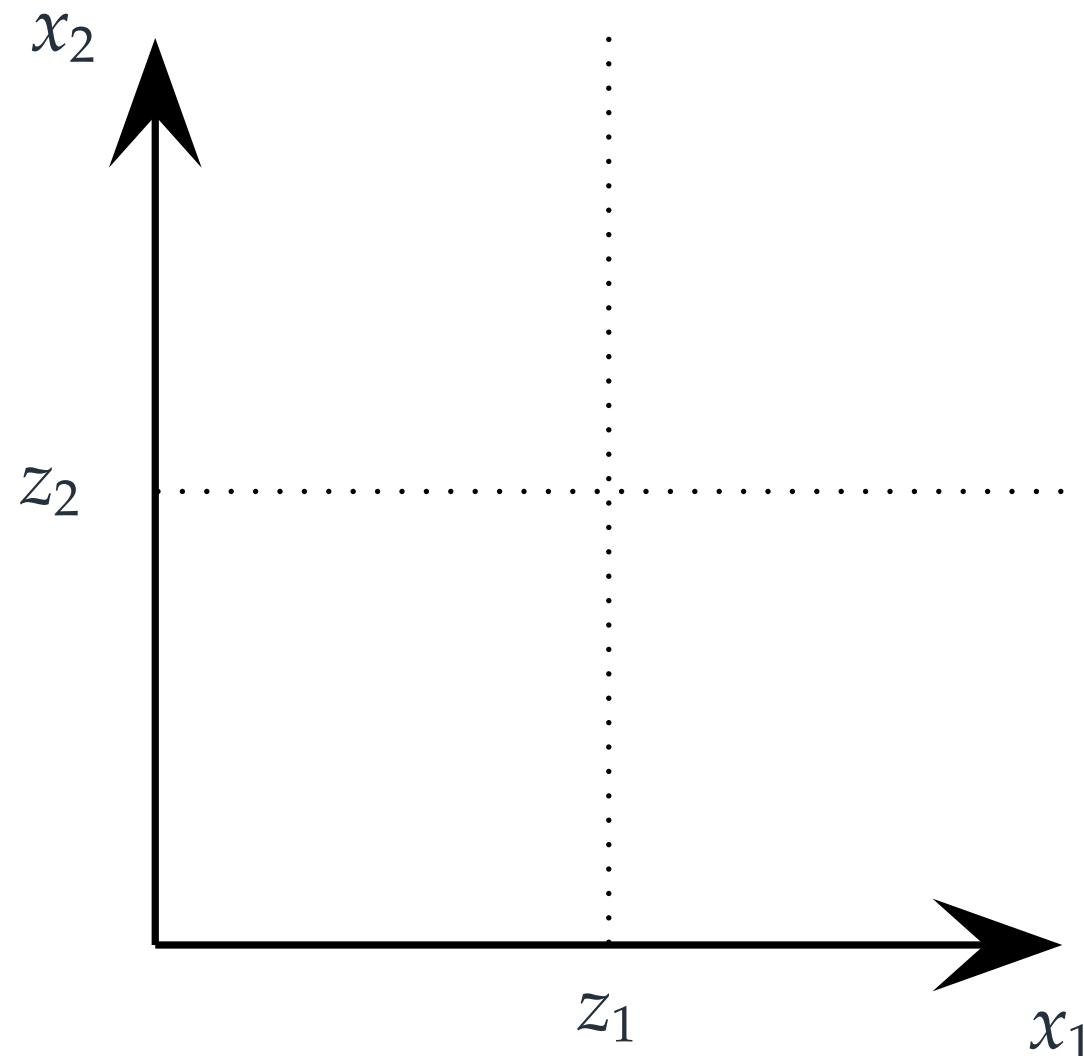


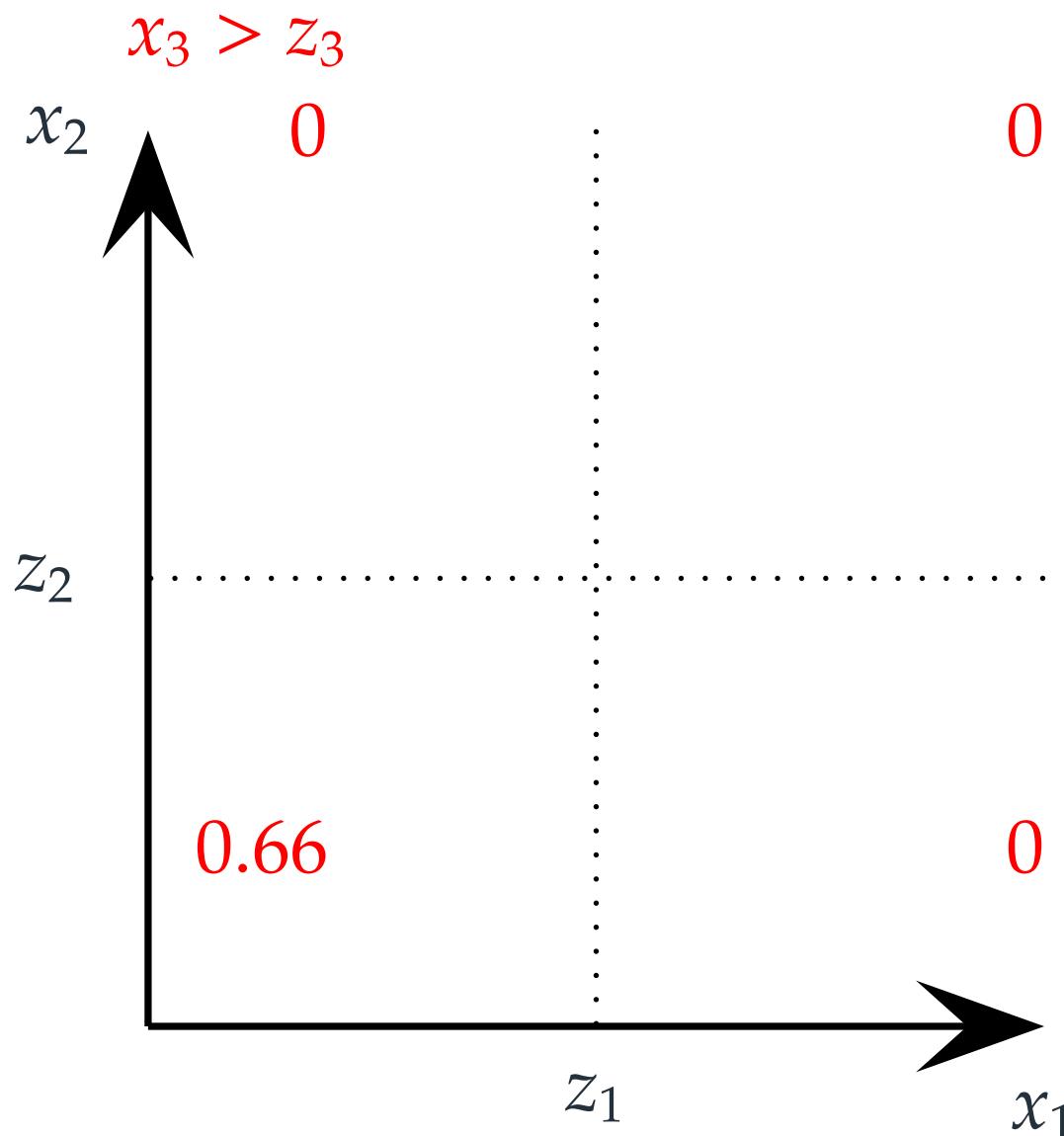
Equalization can increase/decrease MPI

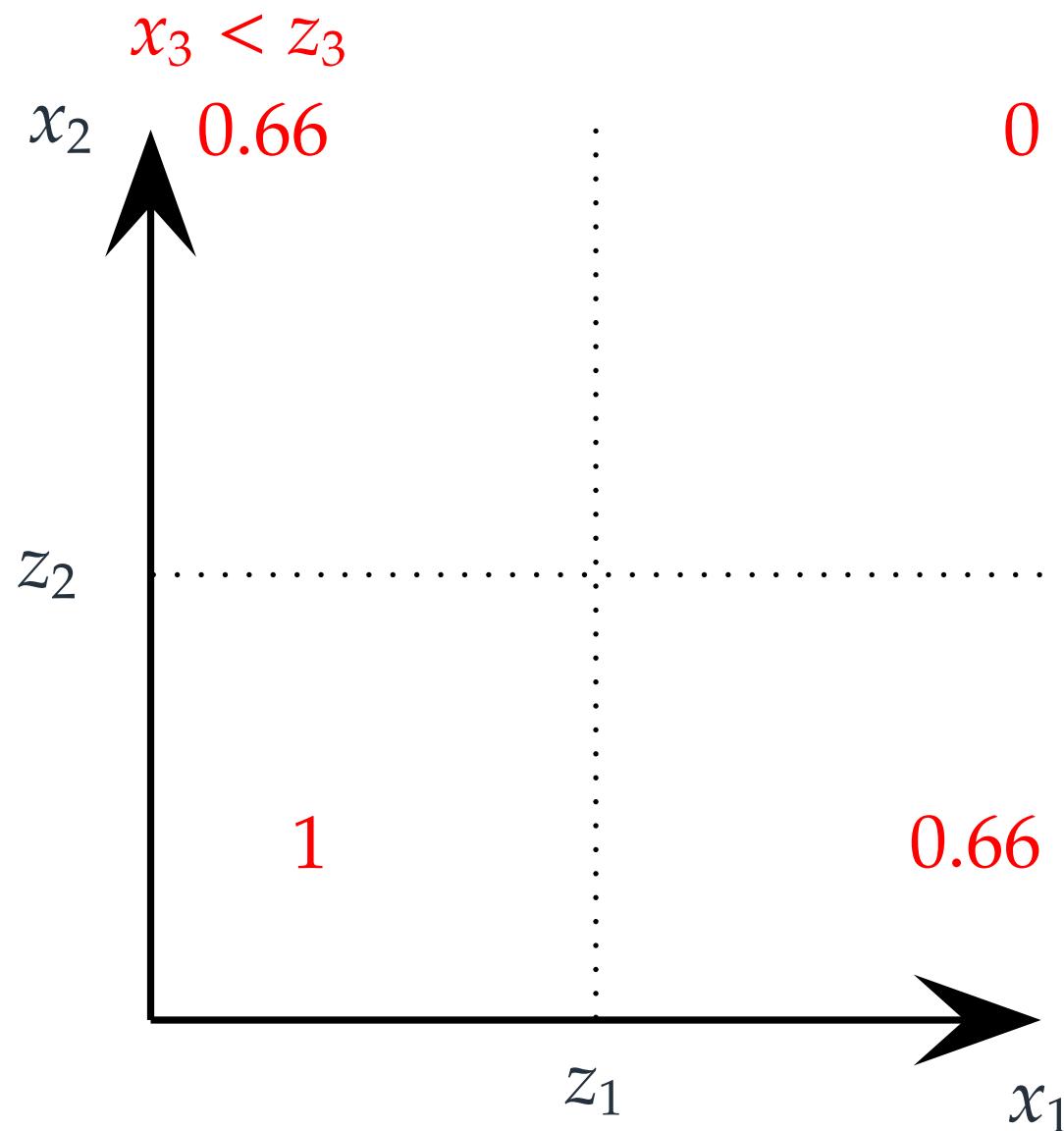


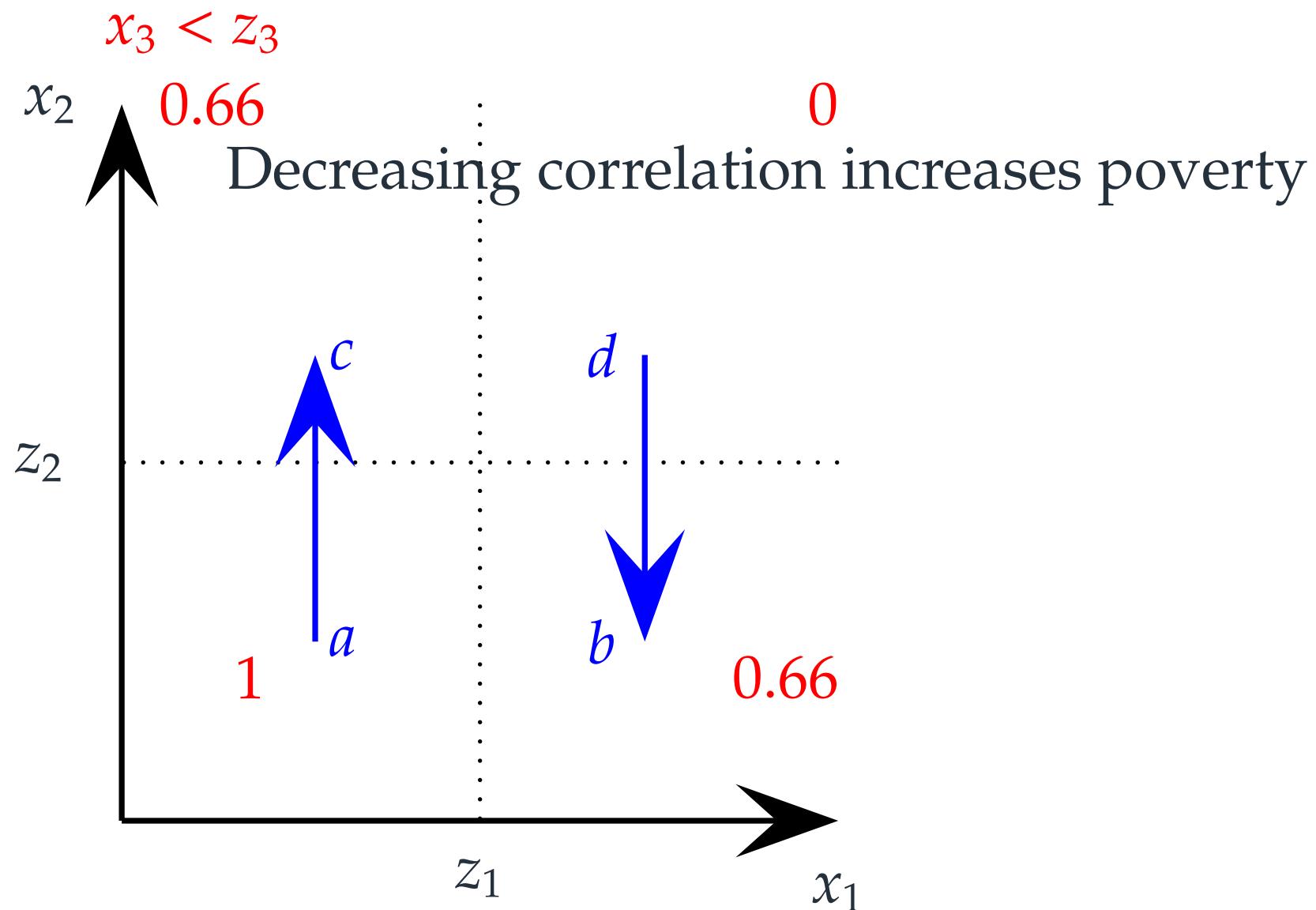
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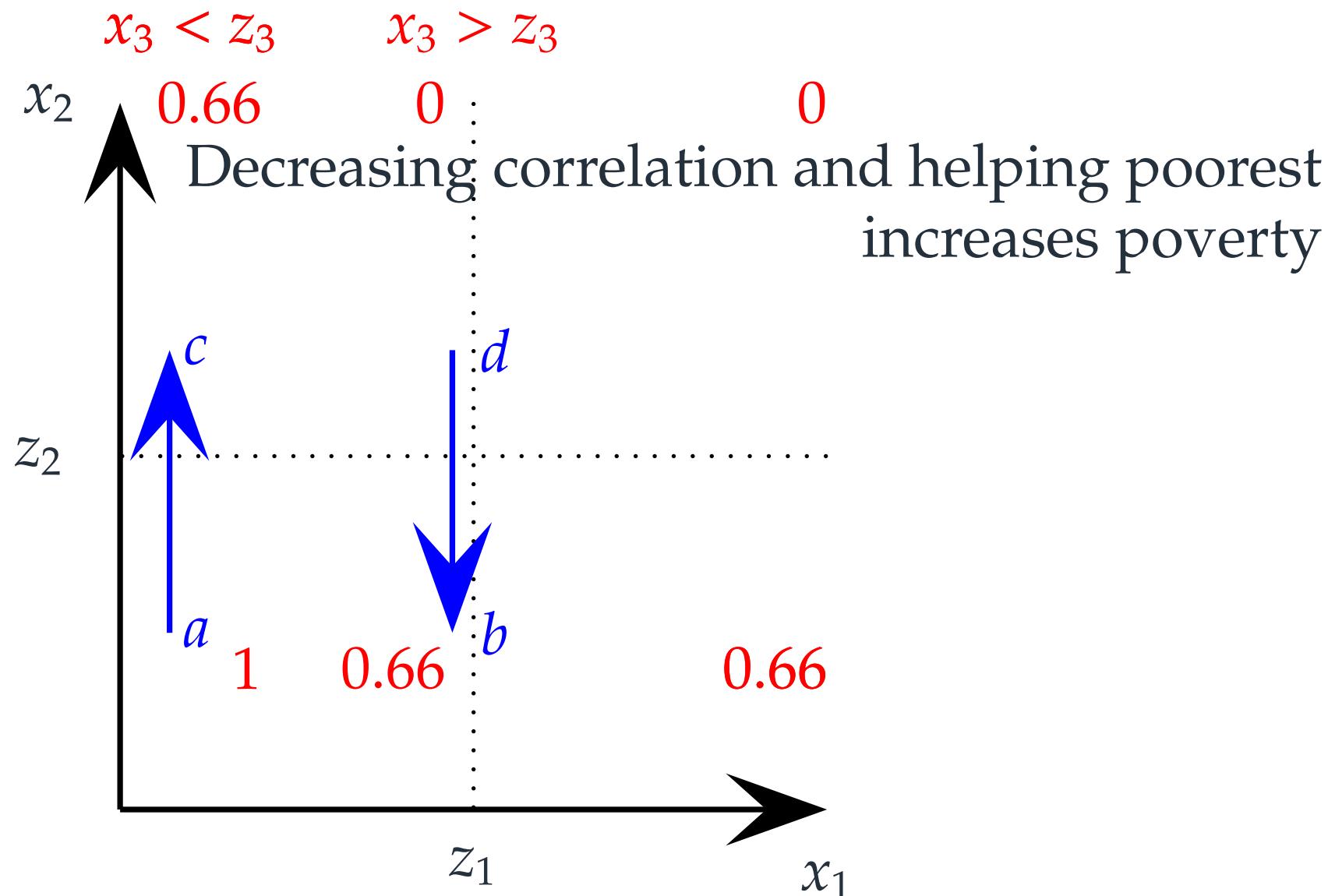
3D discontinuities of $M(2)$ 

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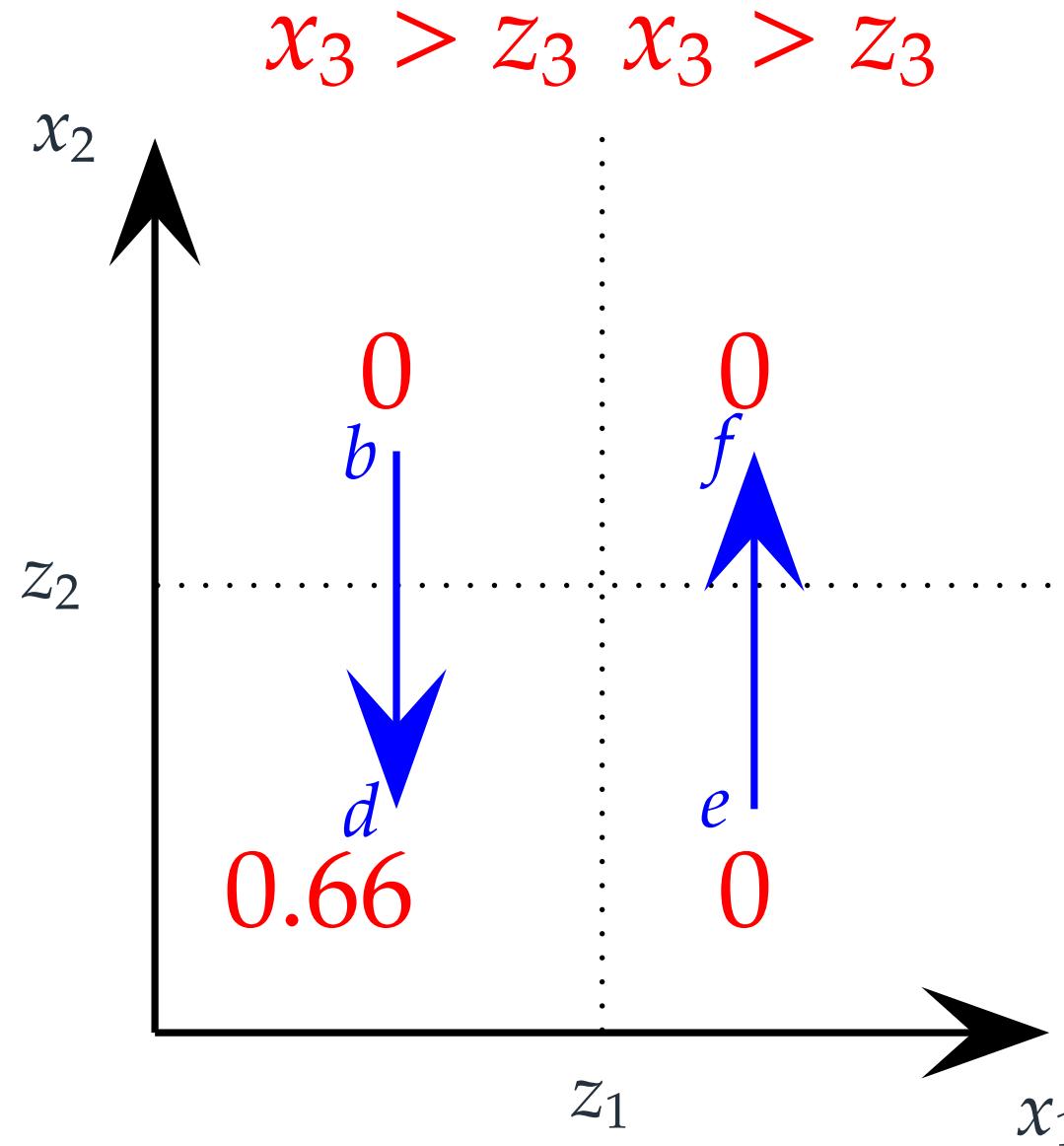
3D discontinuities of $M(2)$ 

3D discontinuities of $M(2)$ 

3D discontinuities of $M(2)$



But increasing correlation can also increase $M(2)$



Outline

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Other indices

Alternative indices

Indices and
properties

Dominance

Other indices

Alternative indices

- UNDP's HDI
- UNDP's HPI
- CMR:

$$\pi(\mathbf{x}_i; \mathbf{z}) = \sum_{k=1}^K \lambda_k g_{ik}^{\alpha}, \quad (4)$$

- Multiplicative FGT:

$$\pi(\mathbf{x}; \mathbf{z}) = \prod_{k=1}^K g_k^{\alpha_k}. \quad (5)$$

- Multi-Watts:

$$\pi(\mathbf{x}; \mathbf{z}) = \sum_{i \in S_k} \sum_{k=1}^K \lambda_k \ln \left[\frac{z_k}{\min(x_k, z_k)} \right], \quad (6)$$

- Tsui:

$$\pi(\mathbf{x}; \mathbf{z}) = \sum_{i=1}^n \lambda_k \left[\prod_{k=1}^K \left(\frac{z_k}{\min(x_k, z_k)} \right)^{\delta_k} - 1 \right], \quad (4)$$

- Datt:

$$\pi(\mathbf{x}; \mathbf{z}) = \left(K^{-1} \sum_{k=1}^K g_k^\alpha \right)^\beta, \quad (5)$$

- Bourguignon-Chakravarty:

$$\pi(\mathbf{x}_i; \mathbf{z}) = \left[\beta_1 g_{i,1}^\varepsilon + (1 - \beta_1) g_{i,2}^\varepsilon \right]^{\frac{\alpha}{\varepsilon}}, \quad (6)$$

Indices and properties

	Con	Foc	Mon	SA	AA	UPDT	MPDT	IPCI
HDI	✓	no	✓	✓	✓	no	no	no
HPI	no	✓	no	✓ (if $\varepsilon = 1$)	✓ (if $\varepsilon = 1$)	no	no	no
UH	no	✓	no	✓	✓	no	no	no
IH	no	✓	no	✓	no	no	no	✓
CMR	✓	✓	✓	✓	✓	✓	✓	no
CDS	✓	✓	✓	✓	✓	✓	✓	no
AF	no	✓	✓ (if $\alpha > 0$)	✓	no	no	no	no
MPI	no	✓	no	✓	no	no	no	no
Tsui	✓	✓	✓	✓	no	no	no	✓
Datt	✓	✓	✓ (if $\alpha, \beta > 0$)	✓	✓ (if $\beta = 1$)	no	$\sqrt{^2}$	✓ (if $\alpha > 0, \beta > 1$)
BC	✓	✓	✓	✓	✓ (if $\alpha = \varepsilon$)	no	no	$\sqrt{^2}$
MFGT	✓	✓	✓	✓	no	no	no	✓

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Sensitivity

“First-order dominance”
indices:

Intersection
headcount

Illustration to
robust (1, 1)
bi-dimensional
poverty in Uganda

Urban minus Rural
Dominance Surface
for Viet Nam

Difference between
1993 and 1998
surfaces for
Ghanaian children

Dominance:
advantages

Dominance:
disadvantages?

Multidimensional poverty

The end

Dominance

Poverty ranking can be reversed

- by a different choice of poverty lines;
- by a different choice of indicator-aggregating procedures;
- by a different choice of individual-aggregating procedures;
- by a different choice of samples.

“First-order dominance” indices:

- Continuity
- Anonymity
- Population
- Monotonicity
- Correlation sensitive

Intersection headcount

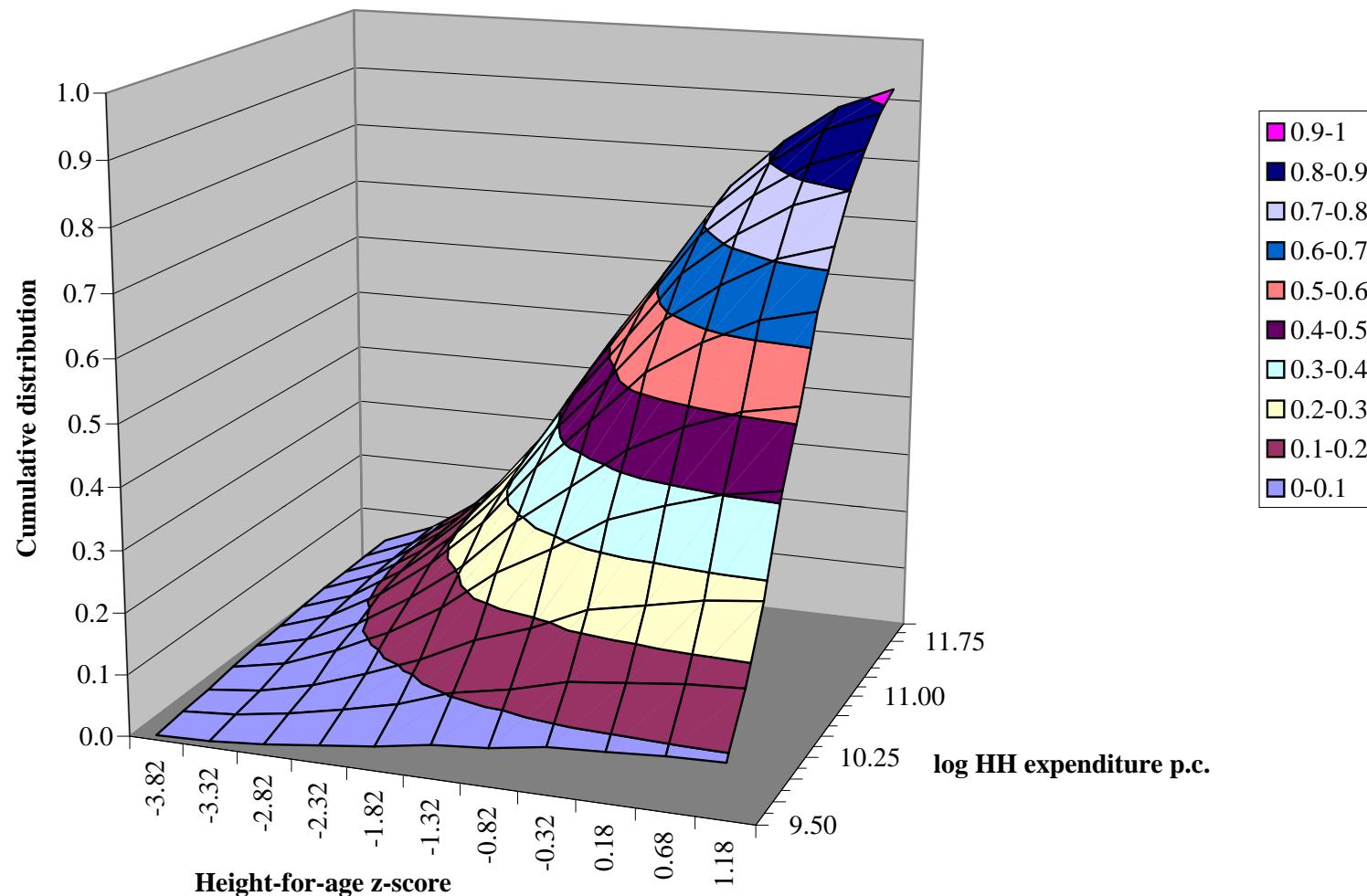


Illustration to robust (1, 1) bi-dimensional poverty in Uganda

Income and education poverty comparisons between Western and Northern regions

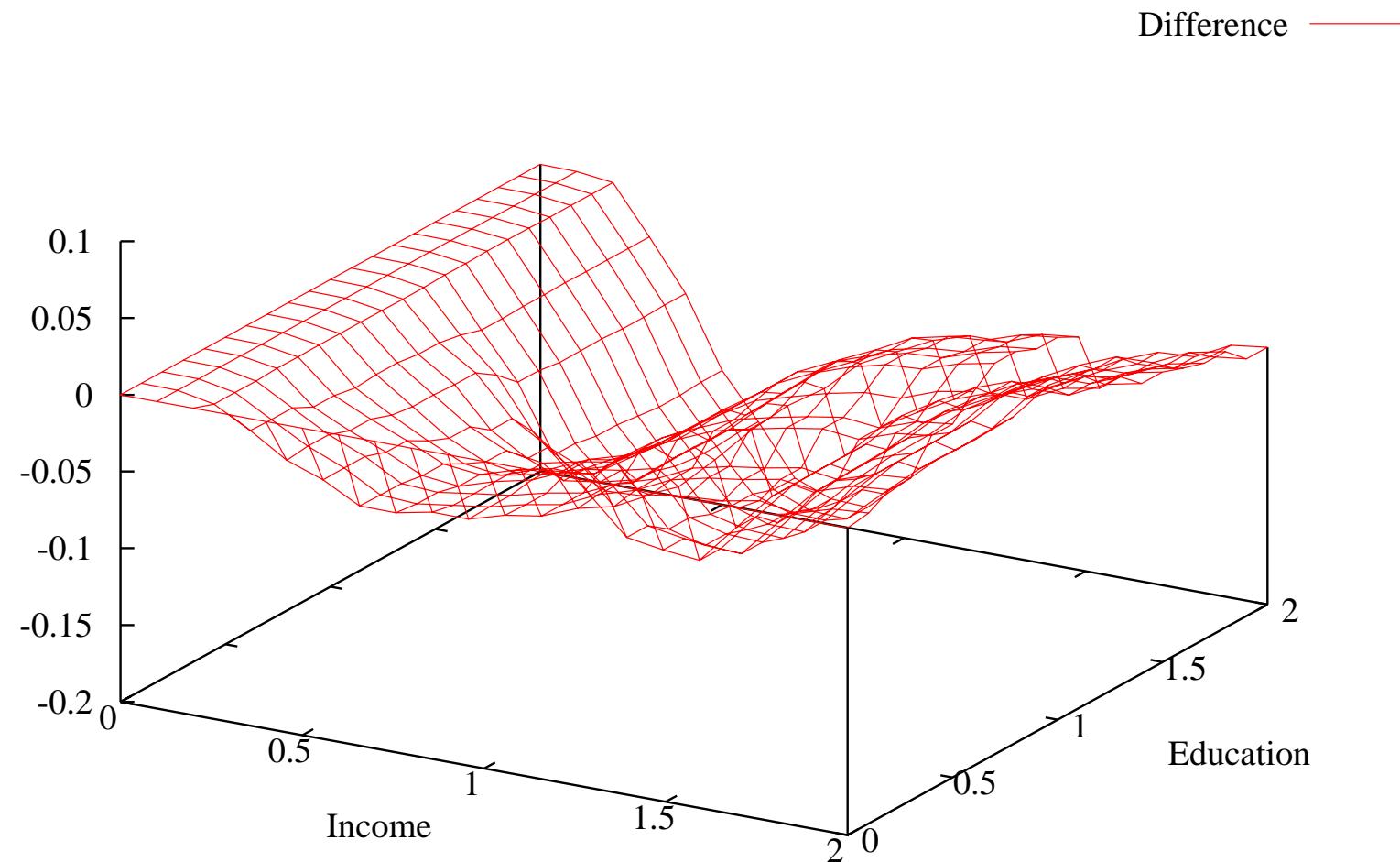
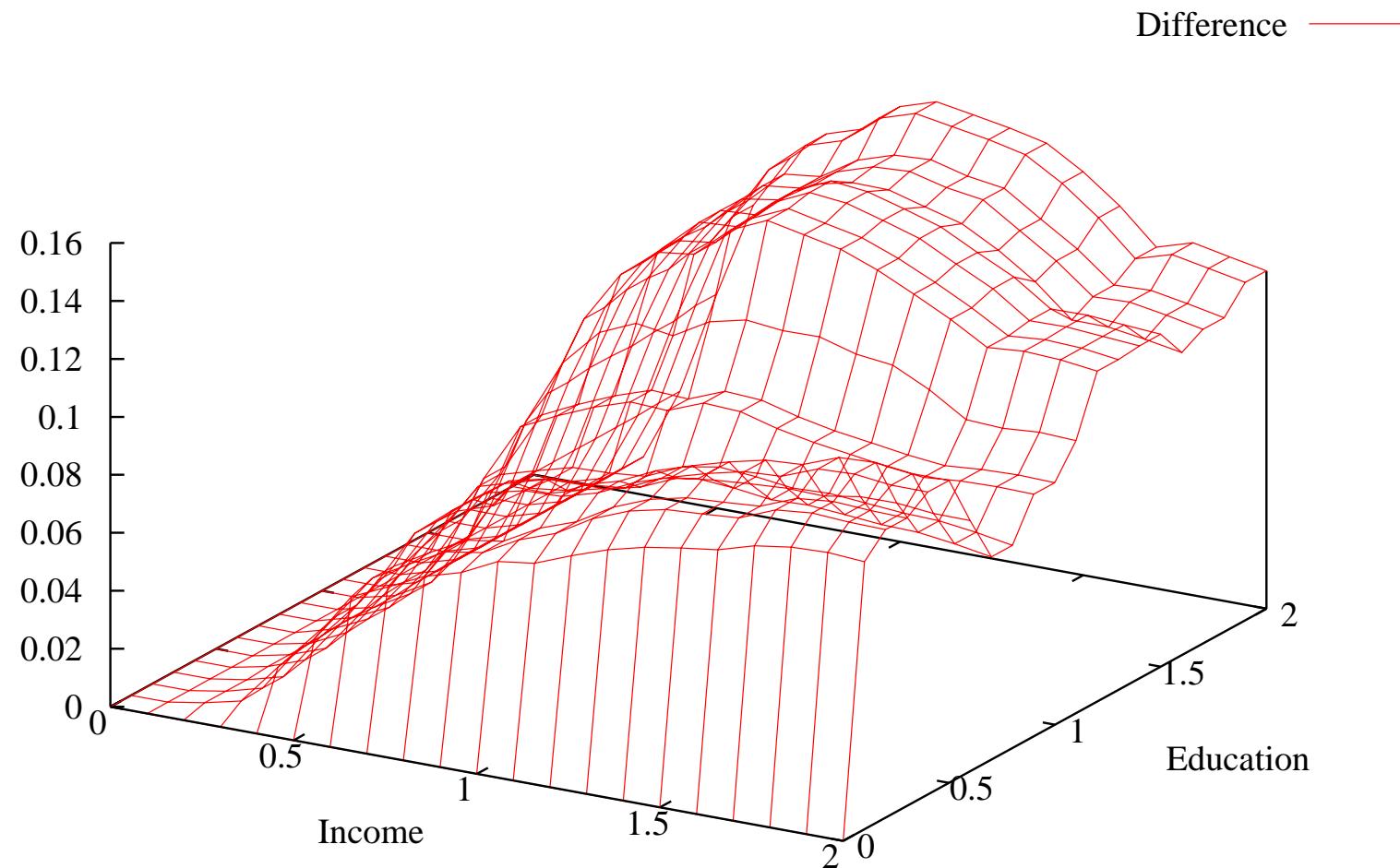
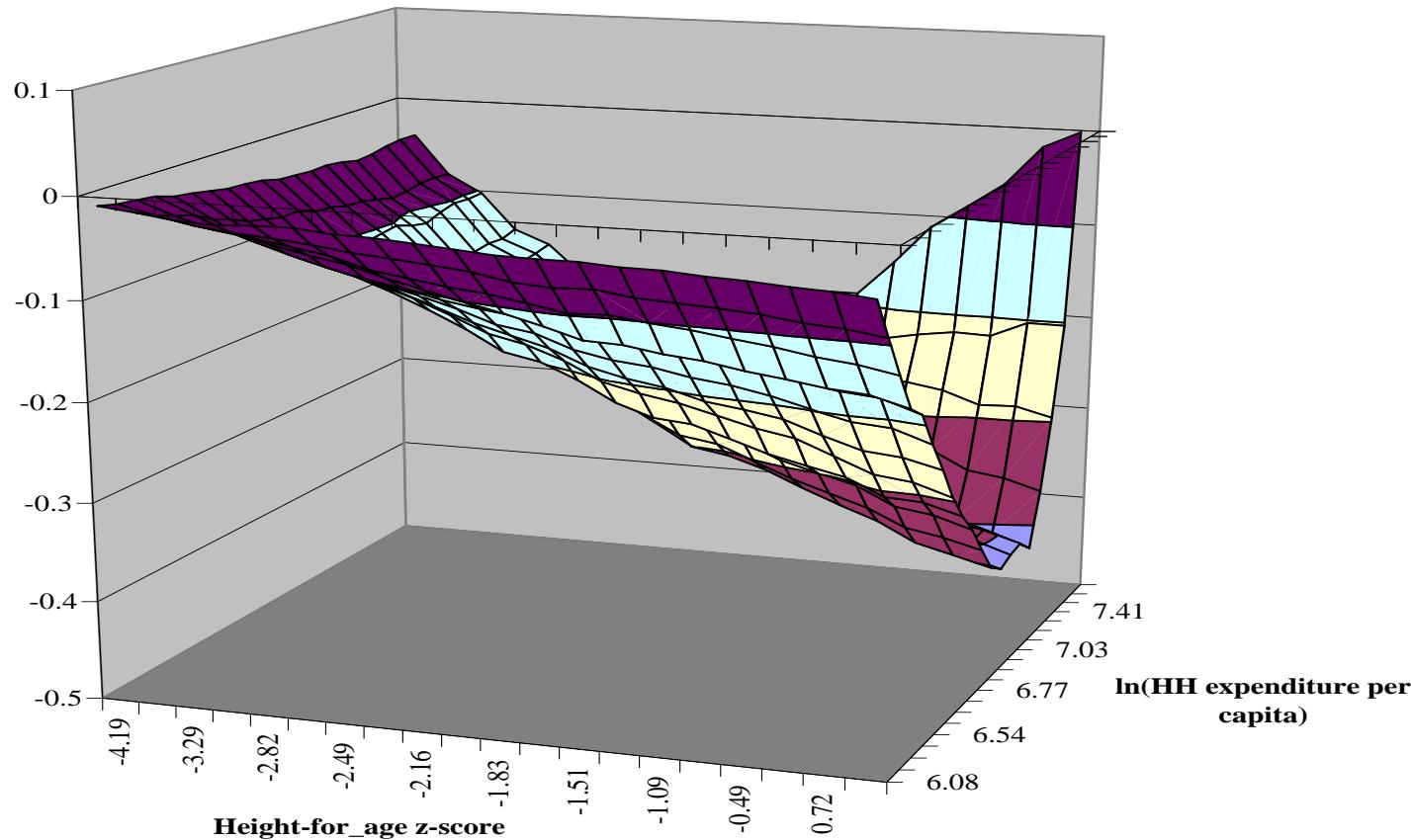


Illustration to robust (1, 1) bi-dimensional poverty in Uganda

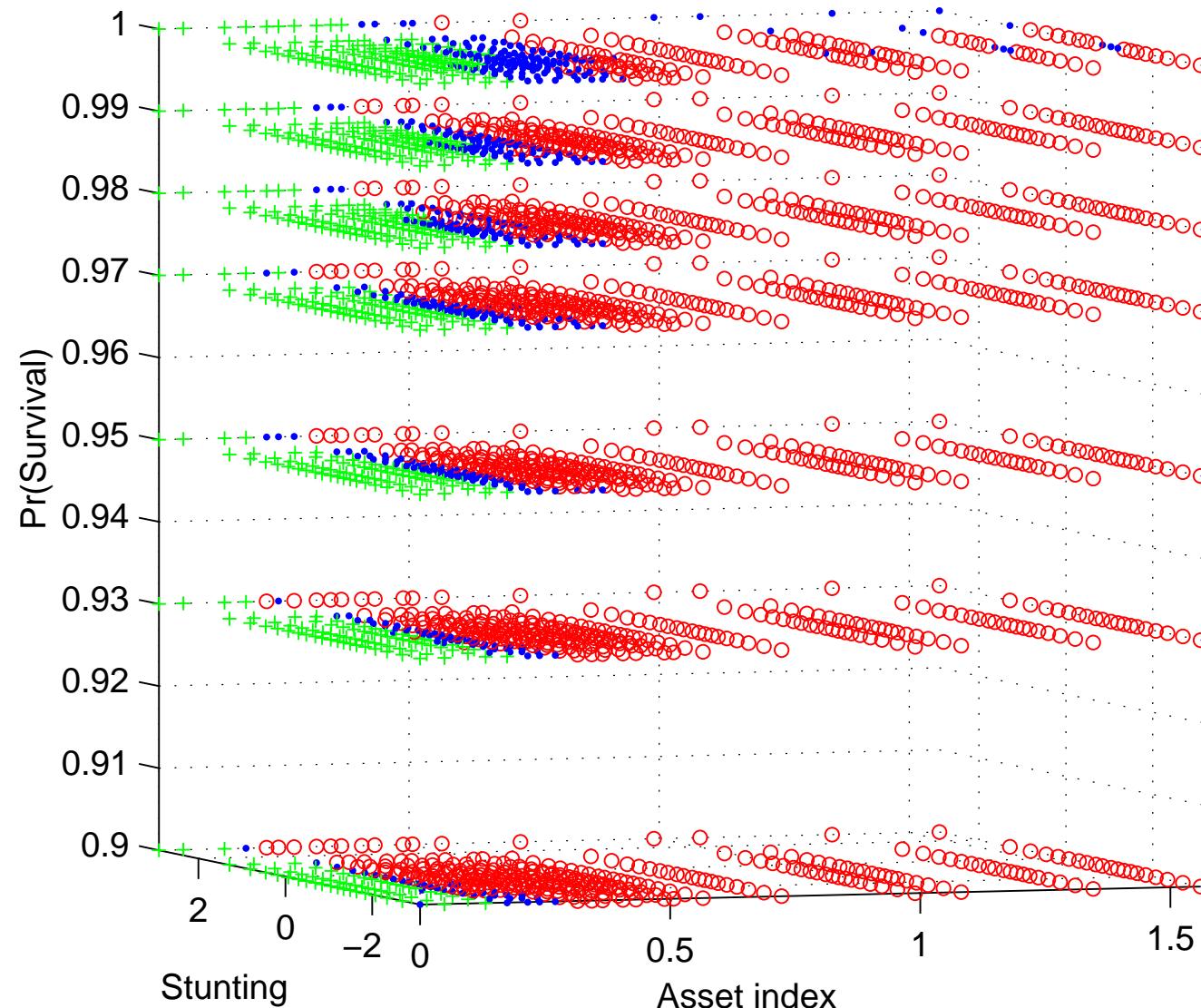
Income and education poverty comparisons between Eastern and Central regions



Urban minus Rural Dominance Surface for Viet Nam



Difference between 1993 and 1998 surfaces for Ghanaian children



Dominance: advantages

- Robust aggregation of dimensions
- Robust aggregation of individuals
- Robust to intersection/union/intermediate types of poverty frontiers
- Robust to position of frontiers
- Appropriate statistical testing techniques now available

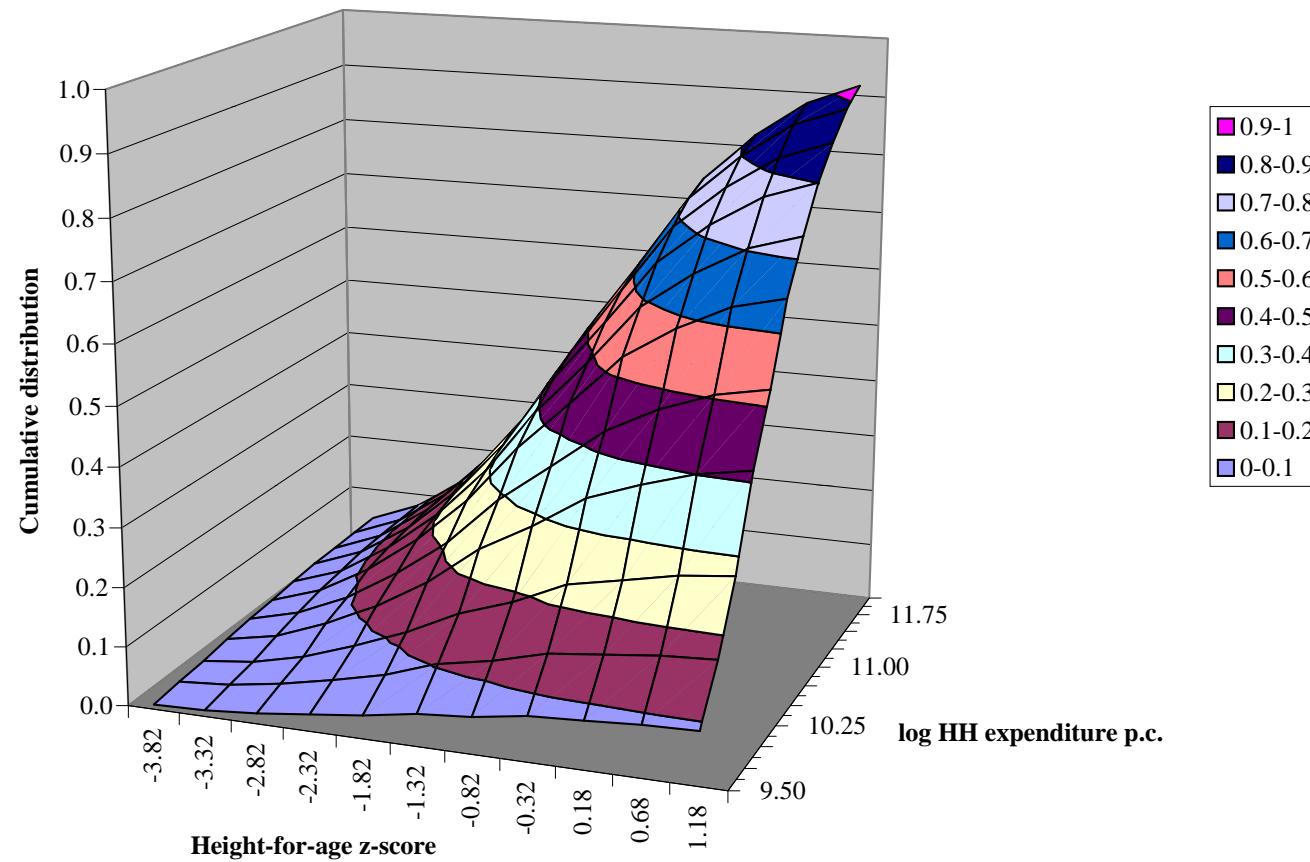
Dominance: disadvantages?

Complicated to do?

- See <http://dasp.ecn.ulaval.ca> for *DASP* or
www.mimap.ecn.ulaval.ca for *DAD*

Dominance: disadvantages?

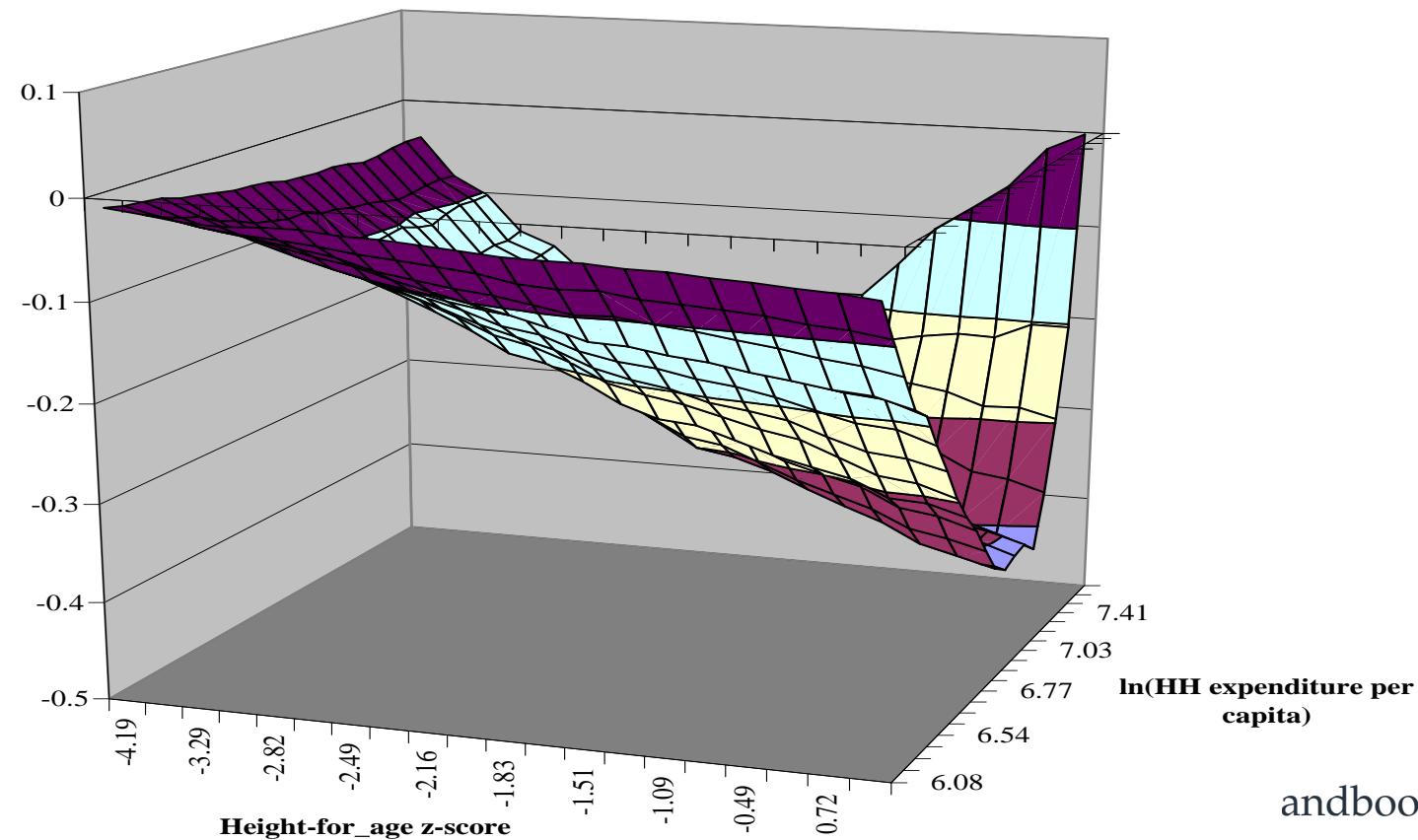
Complicated to understand?



Dominance: disadvantages?

Does not quantify differences?

- Dominance curves are themselves admissible poverty indices



Dominance: disadvantages?

Sometimes inconclusive?

- Can serve to show where disagreements and lack of robustness rest

Dominance: disadvantages?

Table 1: Cross-State Bivariate Dominance Comparisons for Household Assets and Women's Hemoglobin Concentration, India 1999 DHS

	Second-order dominance	
	yes	no
No univariate dominance in either dimension	1	44
Univariate dominance in both dimensions, consistent	90	24
Univariate results inconsistent	34	132
sub-total	125	200

Dominance: disadvantages?

Table 1: Cross-State Bivariate Dominance Comparisons for Household Assets and Student Math Test, Mexico's ENLACE

	Second-order dominance	
	yes	no
No univariate dominance in either dimension	0	21
Univariate dominance in both dimensions, consistent	196	4
Univariate results inconsistent	37	207
sub-total	233	232

Dominance: disadvantages?

Few dimensions allowed?

If not: risks of

- confusion,
- non robustness,
- double counting

Dominance: disadvantages?

Assumes continuity at poverty frontier?

- If not:
 - unattractive properties;
 - lack of robustness;
 - dominance very difficult to establish.

The end

Thank you for your attention.