Social choice under risk and uncertainty

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Philippe Mongin and Marcus Pivato

Centre National de la Recherche Scientifique & HEC Paris

Department of Mathematics, Trent University

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The outcomes of policies are almost always unpredictable.
How should policy-makers cope with such unpredictability?

Economists conventionally distinguish two forms of unpredictability: *risk* and *uncertainty*.

A decision involves *risk* if the possible outcomes can be assigned probabilities in an unambiguous, objective way (e.g. by measuring frequencies in historical data drawn from large, homogeneous population).

Examples:

- Weather forecasting.
- Insurance (property, automobile, workplace, crop, medical, etc.).
- Medical diagnosis, treatment, prognosis.

A decision involves *uncertainty* if the phenomena are *sui generis*, or so poorly understood that we cannot assign clear probabilities to outcomes.

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Decision Theory

(a crash course)

Let $\mathcal{A} := \{a, b, c, \ldots\}$ be a set of outcomes.

A *lottery* is a device $\mathbf{p}$ which assigns a probability to each outcome in $\mathcal{A}$

Formally, $\mathbf{p}$ is a probability distribution: $\mathbf{p} := (p_a, p_b, p_c, \ldots)$.

Let $\mathcal{P} = \{\mathbf{p}^1, \mathbf{p}^2, \mathbf{p}^3, \ldots\}$ be a set of lotteries (e.g. due to different policies).

The problem of *risk*: How to pick the "best" lottery in $\mathcal{P}$.

Let u be a *utility function*, which assigns a "utility" to each outcome in $\mathcal{A}$.

The *expected utility* of a lottery $\mathbf{p}$ is then defined:

$$\mathbb{E}(u, \mathbf{p}) \quad := \quad p_a\, u(a) + p_b\, u(b) + p_c\, u(c) + \cdots$$

The standard way to cope with risk is to choose the lottery which *maximizes expected utility*. **Question:** Is this "rational"?

von Neumann-Morgenstern Theorem. *Let $\Delta(\mathcal{A})$ be the set of all lotteries over $\mathcal{A}$. Let $\succeq$ be a preference order on $\Delta(\mathcal{A})$ which satisfies certain axioms* (describing "consistency" or "rationality"). *Then there is a utility function u on $\mathcal{A}$ such that $\succeq$ seeks to maximize the expected value of u. That is for any lotteries $\mathbf{p}$ and $\mathbf{q}$ in $\Delta(\mathcal{A})$,*

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Risk: Lotteries and von Neumann and Morgenstern

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Let $\mathcal{S} = \{r, s, t, \ldots\}$ be a set of possible "states of the world".

We don't know which is the true state, or even their probabilities.

A *prospect* is a device f which assigns an outcome in $\mathcal{A}$ to each state in $\mathcal{S}$. (Formally, f is a function from $\mathcal{S}$ to $\mathcal{A}$.)

If $f(s) = a$, this means, "If the state of the world turns out to be s, then the prospect will yield the outcome a."

Let $\mathcal{F} = \{f, g, h, \ldots\}$ be a set of prospects (e.g. due to different policies).

The problem of *uncertainty*: How to pick the "best" prospect from $\mathcal{F}$.

A *subjective probability* is a probability distribution p on $\mathcal{S}$. It could represent the "beliefs" of an agent about $\mathcal{S}$.

For instance, if $p(s) > p(t)$, then the agent believes that state s is "more likely" than state t.

Let u be a utility function on the set of outcomes $\mathcal{A}$.

The *subjective expected utility* of prospect f relative to u and p is defined:

$$\mathbb{E}(f|u,p) \quad := \quad p(r)\,u[f(r)] + p(s)\,u[f(s)] + p(t)\,u[f(t)] + \cdots$$

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Savage's Theorem. *Suppose $\mathcal{S}$ is infinite. Let $\mathcal{A}^{\mathcal{S}}$ be the set of all prospects involving $\mathcal{S}$ and $\mathcal{A}$.*
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Uncertainty: Prospects and Savage

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Question: How can we extend the vNM-Savage theory to decision-making for *groups* of people? In effect, this involves two kinds of aggregation:

- Aggregation across different states of nature (i.e. uncertainty).
- Aggregation across different people (i.e. social choice).

Furthermore, social aggregation could be applied either *ex ante* (i.e. *before* the uncertainty is resolved) or *ex post* (i.e. *after* the uncertainty is resolved). The problem is that these two forms of aggregation often clash.

Consider three natural guidelines for social choice under uncertainty:

- **Statewise Dominance principle**: If the lottery/prospect **P** produces a better *ex post* social outcome than **Q** in every state of nature, then **P** is *ex ante* better than **Q**. (Rationality; intertemporal consistency.)
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Prolog Crash course in decision theory.

I Risk: Harsanyi's Social Aggregation Theorem.

II Uncertainty: Heterogeneity and spurious unanimity.

III Equality: The Diamond paradox.

Risk:

Harsanyi's Social Aggregation Theorem

Let $\mathcal{I} := \{\text{Ann, Bob, Carl,} \ldots\}$ be a set of people.

Let $\mathcal{A}$ be a set of social outcomes. Each social outcome in $\mathcal{A}$ yields an outcome for each person in $\mathcal{I}$, determining her consumption bundle, health, autonomy and any other factors relevant to her well-being.

Let $\Delta(\mathcal{A})$ be the space of all lotteries over $\mathcal{A}$ (i.e. "social lotteries").

Each policy corresponds to some element of $\Delta(\mathcal{A})$.

To choose the "best" policy, we need a *social preference order* $\succeq$ on $\Delta(\mathcal{A})$.

If $\succeq$ satisfies the vNM axioms of "rationality", then there is some *ex ante* social welfare function (SWF) W on $\mathcal{A}$ such that $\succeq$ maximizes the expected value of W:

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Then the ex ante *social welfare function W is weighted utilitarian. That is, there exist weights $c_i > 0$ for all individuals i in $\mathcal{I}$, and some constant K, such that, for any outcome a in $\mathcal{A}$, we have*

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Harsanyi and his discontents

- Harsanyi interpreted his SAT as an argument for utilitarianism.
- But Sen argued that Harsanyi's SAT was merely a hollow formal "representation" result, without real ethical content.
- In particular, Sen argued that a vNM utility function conflated an individual's risk preferences with her well-being; hence an aggregate of vNM utilites is not really a measure of social welfare.
- But the SAT is also vulnerable to another serious criticism.
- It assumes a setting of *risk*, where probabilities are *objective*, and known to everyone.
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Social prospects

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(For technical reasons, we also assume that $\mathcal{S}$ is infinite, and p_i and P are *nonatomic* —i.e. $p_i(s) = 0 = P(s)$ for all s in $\mathcal{S}$.)

Let $\mathcal{S}$ be an (infinite) set of states of the world. Let $\mathcal{A}$ be a set of outcomes. Let $\mathcal{A}^{\mathcal{S}}$ be the set of all social prospects (mapping states to outcomes). Let $\mathcal{I}$ be a set of people. For all i in $\mathcal{I}$, let $\succeq_i$ be a preference order on $\mathcal{A}^{\mathcal{S}}$. If $\succeq_i$ satisfies the Savage axioms of "rationality", then there is a utility function u_i on $\mathcal{A}$, and a probability distribution p_i on $\mathcal{S}$ yielding an SEU representation for $\succeq_i$:

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Spurious Unanimity

Statewise Dominance seems non-negotiable. Is *Ex ante* Pareto the culprit?

Indeed, *Ex ante* Pareto is already suspect, for other reasons.

To see this, suppose $\mathcal{S} = \{h, t\}$ and $\mathcal{I} = \{\text{Ann, Bob}\}$, with the beliefs:

	h	t
Ann's probability	0.9	0.1
Bob's probability	0.1	0.9

(i.e. $p_{\text{Ann}}(h) = 0.9$, etc.)

Consider two social prospects $\mathbf{X}$ and $\mathbf{Y}$, with payoffs defined as follows:

$\mathbf{X} :=$

	h	t
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$\mathbf{Y} :=$

	h	t
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Let $\mathfrak{B}$ be the set of events on whose probabilites all agents *agree*. (Formally $\mathfrak{B} := \{\mathcal{E} \subseteq \mathcal{S};\ \ p_i[\mathcal{E}] = p_j[\mathcal{E}]$, for all i and j in $\mathcal{I}\}$.)

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Then $\succeq$ satisfies the restricted ex ante *Pareto condition $\iff$ W is a weighted utilitarian sum of the utilities $\{u_i\}$, and P is a weighted average of the probabilities $\{p_i\}$.*

This seems like a perfect solution. It does *not* require probability agreement, and it is *not* susceptible to spurious unanimity. .... Or is it?

Suppose $\mathcal{S} = \{r, s, t\}$ and $\mathcal{I} = \{\text{Ann, Bob}\}$.

Assume two prospects, f and g, which yield the *same* payoff for both agents in each state of nature.

	r	s	t
f	100	0	100
g	0	100	0

Ann and Bob begin with the *same* prior probability p:

$$p(r) = 0.49, \quad p(s) = 0.02, \quad \text{and} \quad p(t) = 0.49.$$

Ann privately observes the event $\{r, s\}$, while Bob privately observes $\{s, t\}$.

After Bayesian updating, they have the following posterior probabilities:

	Info	r	s	t
Prior		0.49	0.02	0.49
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$$p(r) = 0.49, \quad p(s) = 0.02, \quad \text{and} \quad p(t) = 0.49.$$

Ann privately observes the event $\{r, s\}$, while Bob privately observes $\{s, t\}$.

After Bayesian updating, they have the following posterior probabilities:

	Info	r	s	t
Prior		0.49	0.02	0.49
Ann	{r,s}	0.96	0.04	0
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Theorem. (GSS) *Let W be an* ex post *SWF on $\mathcal{A}$, let P be a probability on $\mathcal{S}$, and let $\succeq$ be the* ex ante *preference relation on $\mathcal{A}^{\mathcal{S}}$ which maximizes the P-expected value of W.*
Then $\succeq$ satisfies the restricted ex ante *Pareto condition $\iff$ W is a weighted utilitarian sum of the utilities $\{u_i\}$, and P is a weighted average of the probabilities $\{p_i\}$.*

This seems like a perfect solution. It does *not* require probability agreement, and it is *not* susceptible to spurious unanimity. .... Or is it?

Suppose $\mathcal{S} = \{r, s, t\}$ and $\mathcal{I} = \{\text{Ann, Bob}\}$.

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GSS attempt to distinguish between "legitimate" unanimity and "spurious" unanimity by an *endogenous* criterion: the agreement set $\mathfrak{B}$.

But this attempt fails. Maybe instead we should use an *exogenous* criterion.

Idea: We should distinguish between *objective* randomness (i.e. "risk") and *subjective* randomness (arising from "uncertainty").

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Equality:

The Diamond Paradox

Statewise Dominance is the "minimal" criterion for rational choice.

But Statewise Dominance can contradict our intuitions about fairness.

Suppose $\mathcal{S} = \{h, t\}$ and $\mathcal{I} = \{\text{Ann,Bob}\}$. Suppose F and G are social prospects that yield state-contingent payoffs as follows (Diamond, 1967):

F	h	t
Ann	1	0
Bob	0	1

G	h	t
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(Canonical example: allocating a hard candy to one of two children.)

Let $\succeq_{\text{xp}}$ be an impartial *ex post* social preference order. So $\begin{bmatrix}1\\0\end{bmatrix} \approx_{\text{xp}} \begin{bmatrix}0\\1\end{bmatrix}$.

Thus, $F(h) \approx_{\text{xp}} G(h)$ and $F(t) \approx_{\text{xp}} G(t)$.

Thus, Statewise Dominance implies that $F \approx_{\text{xa}} G$.

But this contradicts our intuition: F is *fair*, while G is *unfair*.

(*Note:* This paradox does not depend on SEU or subjective beliefs.)

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But Statewise Dominance can contradict our intuitions about fairness.
Suppose $\mathcal{S} = \{h, t\}$ and $\mathcal{I} = \{\text{Ann,Bob}\}$. Suppose F and G are social prospects that yield state-contingent payoffs as follows (Diamond, 1967):

F	h	t
Ann	1	0
Bob	0	1

G	h	t
Ann	1	1
Bob	0	0

(Canonical example: allocating a hard candy to one of two children.)

Let $\succeq_{\rm xp}$ be an impartial *ex post* social preference order. So $\begin{bmatrix}1\\0\end{bmatrix} \approx_{\rm xp} \begin{bmatrix}0\\1\end{bmatrix}$.

Thus, $F(h) \approx_{\rm xp} G(h)$ and $F(t) \approx_{\rm xp} G(t)$.

Thus, Statewise Dominance implies that $F \approx_{\rm xa} G$.

But this contradicts our intuition: F is *fair*, while G is *unfair*.

(*Note:* This paradox does not depend on SEU or subjective beliefs.)

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Our ethical intuitions in Diamond's Paradox exemplify *ex ante* egalitarianism —that is, applying egalitarian principles to the *expected utilities* of the agents, *before* the resolution of uncertainty.

Formally, this means comparing social prospects by applying an (impartial, egalitarian) *ex ante* social welfare function W_{xa} to the ensemble of expected utilities of the agents.

For simplicity, suppose Probability$(h) = \frac{1}{2} =$Probability(t).

Let $EU_A(F)$:=expected utility of Ann in prospect F, etc.

Thus, $EU_A(F) = \frac{1}{2} = EU_B(F)$, whereas $EU_A(G) = 1$ and $EU_B(G) = 0$.

Thus, $F \succ_{\mathrm{xa}} G$ because

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This is to be contrasted with *ex post* egalitarianism, which applies egalitarian principles to social outcomes, *after* the resolution of uncertainty.

Formally, this means comparing social prospects by applying an (impartial, egalitarian) *ex post* social welfare function W_{xp} to each social outcome, and then computing the expected value of W_{xp}.

We have $W_{\mathrm{xp}}\begin{pmatrix}1\\0\end{pmatrix} = C = W_{\mathrm{xp}}\begin{pmatrix}0\\1\end{pmatrix}$ for some constant C.

Thus, $F \approx_{\mathrm{xa}} G$ because

$$\frac{1}{2}W_{\mathrm{xp}}[F(h)] + \frac{1}{2}W_{\mathrm{xp}}[F(t)] = \frac{1}{2}C + \frac{1}{2}C = \frac{1}{2}W_{\mathrm{xp}}[G(h)] + \frac{1}{2}W_{\mathrm{xp}}[G(t)].$$

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In Diamond's Paradox, *ex post* egalitarianism gets the "wrong" answer.

This suggests that *ex ante* egalitarianism is superior.

But consider the following three social prospects:

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E	h	t
Ann	$^1/_2$	$^1/_2$
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E and H yield more egalitarian outcomes than F in *both* states of nature.

So *ex post* egalitarianism would prefer both E and H over F.

(Meanwhile, the choice between E and H might be based on social risk aversion.)

But all three prospects yield the same expected utility ($^1/_2$) for each agent.

So *ex ante* egalitarianism would be indifferent between them.

In this case, it is *ex ante* egalitarianism which gets the "wrong" answer.

Furthermore, XAE and XPE each suffer from a critical flaw.

- XAE violates the Statewise Dominance axiom (so it is "irrational").
- XPE violates the *ex ante* Pareto axiom (so it is "paternalistic").

It seems that *neither* approach is really optimal. Instead, we should employ a "hybrid" of XAE and XPE (Ben Porath, Gilboa & Schmeidler; Gajdos & Maurin).

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He proposed to rank such prospects by the expected value of an *ex post* SWF W_{xp} having the property: $W_{\mathrm{xp}}(u, u, \ldots, u) = u$, for any u in $\mathbb{R}$.

He called these *expected equally distributed equivalent* (EEDE) rankings.

An *egalitarian prospect* is one which yields a perfectly egalitarian social outcome in every state. For example:

F	s_1	s_2	s_3	s_4	s_5
Ann	3	0	5	2	-1
Bob	3	0	5	2	-1
Carol	3	0	5	2	-1

If F is egalitarian, then Expected Value(W_{xp}, F)=Expected Utility$_i$(F) for every individual i in $\mathcal{I}$ (assuming common probabilistic beliefs).

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Any XPE ranking satisfies *ex ante* Pareto if comparing riskless prospects.

And any XPE ranking satisfies Statewise Dominance.

In fact, these three properties characterise the EEDE rankings.

Theorem. (Fleurbaey, 2010) *Let $\succeq$ be a ranking of all social prospects. Then $\succeq$ satisfies Statewise Dominance,* ex ante *Pareto for riskless prospects, and* ex ante *Pareto for egalitarian prospects, if and only if $\succeq$ is an EEDE ranking.*

By extending *ex ante* Pareto to "non-reranking prospects", Fleurbaey also characterized two subclasses of EEDE rankings: *ex post generalized Gini* and *ex post leximin.*

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Social choice under uncertainty becomes surprisingly complicated, once issues of fairness and heterogeneity come into play.

However, there are also other problems we haven't even discussed...

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Conclusion

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Thank you.

Appendix A

Sen's objections to Harsanyi's SAT

Harsanyi's Social Aggregation Theorem: *If $\succeq_i$ all satisfy* **Individual vNM** *and $\succeq$ satisfies* **Group vNM** *and* **XAP**, *then there are weights $c_i > 0$ for all i in $\mathcal{I}$, and a constant K, such that, for any a in $\mathcal{A}$, we have*

$$W(a) \quad = \quad K + c_{\mathrm{Ann}}\, u_{\mathrm{Ann}}(a) + c_{\mathrm{Bob}}\, u_{\mathrm{Bob}}(a) + c_{\mathrm{Carl}}\, u_{\mathrm{Carl}}(a) + \cdots$$

Harsanyi interpreted his SAT as an argument for utilitarianism.
But Sen made three objections to this interpretation....

(S1) The SAT just says that any $\succeq$ satisfying **Group vNM** and **XAP** can be interpreted *as if* it came from some weighted utilitarian social welfare function, for some suitable choice of weights $c_{\mathrm{Ann}}, c_{\mathrm{Bob}}, c_{\mathrm{Carl}}, \ldots$
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(S2) The vNM utility functions u_i are only defined up to multiplication by a constant: If u_i represents $\succeq_i$, then so does $3 \cdot u_i$.
Thus, the weights c_i are *also* only defined up to multiplication by a constant. (If we replace u_{Ann} with $3\, u_{\mathrm{Ann}}$, then we replace c_{Ann} with $c_{\mathrm{Ann}}/3$.)

(S3) u_i represents person i's preferences over *lotteries*. But it isn't clear u_i really measures i's *well-being*. So W is not really "utilitarian" —it is an aggregate of "lottery preferences", not an aggregate of well-being *per se*.

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It merely invalidates Harsanyi's "utilitarian" interpretation of the SAT.

"**(S3)** u_i represents person i's preferences over *lotteries*. But it isn't clear u_i really measures i's *well-being*. So W is not really 'utilitarian' —it is an aggregate of 'lottery preferences', not an aggregate of well-being *per se*."

To appreciate this, suppose w_i is person i's "true" welfare function. (So $w_i(a)=$ the welfare of person i in outcome a.)
Assuming i is self-interested, her vNM utility function u_i should have the form $u_i(a) = \phi_i[w_i(a)]$, where ϕ_i is some increasing function.
The shape of ϕ_i expresses i's "risk attitudes". For example:

- if i is *risk-averse*, then ϕ_i is concave.
- If i is *risk-prone* then ϕ_i is convex.

But *there is no reason to suppose that ϕ_i is linear.*
Thus, Harsanyi's SAT actually characterizes an *ex ante* SWF of the form:

$$W(a) = K + c_{\mathrm{Ann}}\,\phi_{\mathrm{Ann}}[w_{\mathrm{Ann}}(a)] + c_{\mathrm{Bob}}\,\phi_{\mathrm{Bob}}[w_{\mathrm{Bob}}(a)] + c_{\mathrm{Carl}}\,\phi_{\mathrm{Carl}}[w_{\mathrm{Carl}}(a)] + \cdots$$

This does not invalidate SAT's analysis of social choice under uncertainty.
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"**(S1)** The SAT says $\succeq$ can be interpreted *as if* it came from some weighted utilitarian SWF... But it doesn't tell us how to *obtain* these weights.
(S2) The vNM utility functions u_i are only defined up to multiplication by a constant. Thus, the weights c_i are *also* only defined up to multiplication by a constant. "

One solution is to supplement the SAT with a rule to "normalize" the vNM utility functions.

For example, Dhillon and Mertens axiomatically characterized the *relative utilitarian ex ante* social welfare function RU, defined

$$RU(a) \quad := \quad \frac{u_{\mathrm{Ann}}(a)}{R_{\mathrm{Ann}}} + \frac{u_{\mathrm{Bob}}(a)}{R_{\mathrm{Bob}}} + \frac{u_{\mathrm{Carl}}(a)}{R_{\mathrm{Carl}}} + \cdots \quad \text{for all } a \text{ in } \mathcal{A}.$$

Here, for all i in $\mathcal{I}$, $R_i := \max\{u_i(a);\ a \in \mathcal{A}\} - \min\{u_i(a);\ a \in \mathcal{A}\}$.
Thus, for all i in $\mathcal{I}$, the function u_i/R_i ranges over an interval of length 1.
So everyone has equal "weight", in some sense.
But it's not clear this renormalization is ethically appropriate.
For example, there may be a legitimate reason why Ann has much more intense preferences over $\mathcal{A}$ than Bob does. Perhaps she has a much greater stake in the outcomes. Relative utilitarianism neglects these differences.

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Appendix B

Bayesian social aggregation with minimal hypotheses

Let $\mathcal{S}$ be a finite set of states.

We now define an *individual prospect* to be a device $\mathbf{x}$ which assigns a real-valued "payoff" x_s (e.g. an income or utility level) to each state $s \in S$. (Formally, this means that $\mathbf{x}$ is an $\mathcal{S}$-dimensional vector of real numbers.)

Let $\mathcal{I}$ be a set of individuals. For all $i \in \mathcal{I}$, let $\mathcal{X}^i$ be the set of individual prospects which are feasible for i (a subset of $\mathbb{R}^{\mathcal{S}}$).
Let $\succeq^i$ be individual i's *ex ante* preference order on $\mathcal{X}^i$.

Instead of the Savage SEU axioms, we will now only require:

Individual statewise dominance: For all $\mathbf{x}$ and $\mathbf{y}$ in $\mathcal{X}^i$, if $x_s \geq y_s$ for all s in $\mathcal{S}$, then $\mathbf{x} \succeq^i \mathbf{y}$. If, also, $x_s > y_s$ for some s in $\mathcal{S}$, then $\mathbf{x} \succ^i \mathbf{y}$.

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Ex post Pareto: For all outcomes $\mathbf{x}$ and $\mathbf{y}$ in $\mathcal{X}_{\mathrm{xp}}$, if $x^i \geq y^i$ for all i in $\mathcal{I}$, then $\mathbf{x} \succeq_{\mathrm{xp}} \mathbf{y}$. If, also, $x^i > y^i$ for some i in $\mathcal{I}$, then $\mathbf{x} \succ_{\mathrm{xp}} \mathbf{y}$.

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Finally, we will define a *social prospect* to be a device $\mathbf{X}$ which assigns a social outcome $\mathbf{x}_s$ to each state $s \in S$.

Equivalently, $\mathbf{X}$ assigns an individual prospect $\mathbf{x}^i$ to each person i in $\mathcal{I}$.

Equivalently, $\mathbf{X}$ yields a real-valued "payoff" x_s^i to all i in $\mathcal{I}$, for all s in $\mathcal{S}$. (Formally, this means $\mathbf{X}$ is an $\mathcal{I} \times \mathcal{S}$ matrix of real numbers).

Let $\mathcal{X}$ be the set of feasible individual prospects (a subset of $\mathbb{R}^{\mathcal{I} \times \mathcal{S}}$).

Let $\succeq_{\text{xa}}$ be the *ex ante* social preference order on $\mathcal{X}$. We will require

For all social prospects $\mathbf{X}$ and $\mathbf{Y}$ in $\mathcal{X}$:

- **Ex ante Pareto:** If $\mathbf{x}^i \succeq^i \mathbf{y}^i$ for all i in $\mathcal{I}$, then $\mathbf{X} \succeq_{\text{xa}} \mathbf{Y}$.
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Theorem. (M & P, 2013, extending Blackorby, Donaldson & Mongin, 2004) *Let $\mathcal{X}$ be an open, connected subset of $\mathbb{R}^{\mathcal{I}\times\mathcal{S}}$ (also with two other connectivity conditions). For all i in $\mathcal{I}$, let $\succeq^i$ be individual i's* ex ante *preference order on $\mathcal{X}^i$. Let $\succeq_{\rm xp}$ be an* ex post *social preference order on $\mathcal{X}_{\rm xp}$. Suppose*

- *either $\succeq_{\rm xp}$ satisfies* **Ex post Pareto***;*
- *or $\succeq^i$ satisfies* **Individual Statewise Dominance***, for all i in $\mathcal{I}$.*

Finally, let $\succeq_{\rm xa}$ be an ex ante *social preference order on $\mathcal{X}$.*
If $\succeq_{\rm xa}$ satisfies **Continuity**, **Group Statewise Dominance**, *and* **Ex ante Pareto**, *then there is a (unique) probability P on $\mathcal{S}$, and for all i in $\mathcal{I}$, there are (unique) increasing, continuous utility functions u^i, such that:*
(a) *For all i in $\mathcal{I}$, the order $\succeq^i$ maximizes the P-expected value of u^i.*
(b) *$\succeq_{\rm xp}$ is represented by the utilitarian* ex post *social welfare function $W_{\rm xp}$ defined by $W_{\rm xp}(\mathbf{x}) := \sum_{i\in I} u^i(x^i)$, for all $\mathbf{x} \in \mathcal{X}_{\rm xp}$.*
(c) *The* ex ante *order $\succeq_{\rm xa}$ maximizes the P-expected value of $W_{\rm xp}$.*

Upshot: Even if we weaken the Savage axioms to Statewise Dominance (perhaps the weakest "rationality" axiom imaginable), we *still* get all the conclusions of the Bayesian Social Aggregation Theorem.

The *ex post* social welfare function $W_{\rm xp}$ is utilitarian, all agents are SEU maximizers, and *all agents must have the same beliefs*.

Theorem. (M & P, 2013, extending Blackorby, Donaldson & Mongin, 2004) *Let $\mathcal{X}$ be an open, connected subset of $\mathbb{R}^{\mathcal{I}\times\mathcal{S}}$ (also with two other connectivity conditions). For all i in $\mathcal{I}$, let $\succeq^i$ be individual i's* ex ante *preference order on $\mathcal{X}^i$. Let $\succeq_{\mathrm{xp}}$ be an* ex post *social preference order on $\mathcal{X}_{\mathrm{xp}}$. Suppose*

- *either $\succeq_{\mathrm{xp}}$ satisfies* **Ex post Pareto***;*
- *or $\succeq^i$ satisfies* **Individual Statewise Dominance***, for all i in $\mathcal{I}$.*

Finally, let $\succeq_{\mathrm{xa}}$ be an ex ante *social preference order on $\mathcal{X}$.*
If $\succeq_{\mathrm{xa}}$ satisfies **Continuity**, **Group Statewise Dominance**, *and* **Ex ante Pareto**, *then there is a (unique) probability P on $\mathcal{S}$, and for all i in $\mathcal{I}$, there are (unique) increasing, continuous utility functions u^i, such that:*
(a) *For all i in $\mathcal{I}$, the order $\succeq^i$ maximizes the P-expected value of u^i.*
(b) *$\succeq_{\mathrm{xp}}$ is represented by the utilitarian* ex post *social welfare function W_{xp} defined by $W_{\mathrm{xp}}(\mathbf{x}) := \sum_{i\in I} u^i(x^i)$, for all $\mathbf{x} \in \mathcal{X}_{\mathrm{xp}}$.*
(c) *The* ex ante *order $\succeq_{\mathrm{xa}}$ maximizes the P-expected value of W_{xp}.*

Upshot: Even if we weaken the Savage axioms to Statewise Dominance (perhaps the weakest "rationality" axiom imaginable), we *still* get all the conclusions of the Bayesian Social Aggregation Theorem.
The *ex post* social welfare function W_{xp} is utilitarian, all agents are SEU maximizers, and *all agents must have the same beliefs.*

Appendix C

Complete specification

in the

Diamond Paradox

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1

G	h	t
Ann	1	1
Bob	0	0

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1

G	h	t
Ann	1	1
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F	h	t
Ann	1	0
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It is important to stipulate that these payoff tables provide a complete description of the problem.
We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

To see this, suppose that, in addition to receiving the stated payoffs from the candy at time 1, each person gets an *anticipation* utility of $p\,\alpha$ at time 0, where p is the probability they will receive the candy at time 1.

Complete specification in the Diamond Paradox

F	h	t
Ann	$1 + \alpha/2$	$\alpha/2$
Bob	$\alpha/2$	$1 + \alpha/2$

G	h	t
Ann	$1 + \alpha$	$1 + \alpha$
Bob	0	0

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

To see this, suppose that, in addition to receiving the stated payoffs from the candy at time 1, each person gets an *anticipation* utility of $p\,\alpha$ at time 0, where p is the probability they will receive the candy at time 1.

Then their *total* payoffs (time 0 + time 1) are as shown in the new table.

Complete specification in the Diamond Paradox

F	h	t
Ann	$1+\alpha/2$	$\alpha/2$
Bob	$\alpha/2$	$1+\alpha/2$

G	h	t
Ann	$1+\alpha$	$1+\alpha$
Bob	0	0

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

To see this, suppose that, in addition to receiving the stated payoffs from the candy at time 1, each person gets an *anticipation* utility of $p\,\alpha$ at time 0, where p is the probability they will receive the candy at time 1.

Then their *total* payoffs (time $0+$ time 1) are as shown in the new table.

Now F yields a more egalitarian outcome than G in both states of nature.

F	h	t
Ann	$1+\alpha/2$	$\alpha/2$
Bob	$\alpha/2$	$1+\alpha/2$

G	h	t
Ann	$1+\alpha$	$1+\alpha$
Bob	0	0

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

To see this, suppose that, in addition to receiving the stated payoffs from the candy at time 1, each person gets an *anticipation* utility of $p\,\alpha$ at time 0, where p is the probability they will receive the candy at time 1.

Then their *total* payoffs (time 0 + time 1) are as shown in the new table.

Now F yields a more egalitarian outcome than G in both states of nature.

So if $\succeq_{\rm xp}$ is any egalitarian *ex post* SWO, then $F(h) \succ_{\rm xp} G(h)$ and $F(t) \succ_{\rm xp} G(t)$. Hence $F(h) \succ_{\rm xa} G(h)$ by Statewise Dominance.

Complete specification in the Diamond Paradox

F	h	t
Ann	$1 + \alpha/2$	$\alpha/2$
Bob	$\alpha/2$	$1 + \alpha/2$

G	h	t
Ann	$1 + \alpha$	$1 + \alpha$
Bob	0	0

It is important to stipulate that these payoff tables provide a complete description of the problem.
We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

To see this, suppose that, in addition to receiving the stated payoffs from the candy at time 1, each person gets an *anticipation* utility of $p\,\alpha$ at time 0, where p is the probability they will receive the candy at time 1.
Then their *total* payoffs (time 0 + time 1) are as shown in the new table.
Now F yields a more egalitarian outcome than G in both states of nature.
So if $\succeq_{\rm xp}$ is any egalitarian *ex post* SWO, then $F(h) \succ_{\rm xp} G(h)$ and $F(t) \succ_{\rm xp} G(t)$. Hence $F(h) \succ_{\rm xa} G(h)$ by Statewise Dominance.
So the Diamond Paradox only works if we exclude “anticipation” effects (Deschamps and Gevers, 1979)

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1

G	h	t
Ann	1	1
Bob	0	0

It is important to stipulate that these payoff tables provide a complete description of the problem.
We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

On the other hand, suppose that Bob felt some "bitterness" $-\beta$ due to his unfair treatment in prospect G.

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1

G	h	t
Ann	1	1
Bob	$-\beta$	$-\beta$

It is important to stipulate that these payoff tables provide a complete description of the problem.
We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

On the other hand, suppose that Bob felt some “bitterness” $-\beta$ due to his unfair treatment in prospect G.

Then his *actual* payoffs would be as shown in the revised table.

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1

G	h	t
Ann	1	1
Bob	$-\beta$	$-\beta$

It is important to stipulate that these payoff tables provide a complete description of the problem.
We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

On the other hand, suppose that Bob felt some "bitterness" $-\beta$ due to his unfair treatment in prospect G.

Then his *actual* payoffs would be as shown in the revised table.

So if $\succeq_{\rm xp}$ is any impartial, Pareto *ex post* SWO, then $F(h) \succ_{\rm xp} G(h)$ and $F(t) \succ_{\rm xp} G(t)$. Hence $F(h) \succ_{\rm xa} G(h)$ by Statewise Dominance.

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1

G	h	t
Ann	1	1
Bob	$-\beta$	$-\beta$

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

On the other hand, suppose that Bob felt some "bitterness" $-\beta$ due to his unfair treatment in prospect G.

Then his *actual* payoffs would be as shown in the revised table.

So if $\succeq_{\rm xp}$ is any impartial, Pareto *ex post* SWO, then $F(h) \succ_{\rm xp} G(h)$ and $F(t) \succ_{\rm xp} G(t)$. Hence $F(h) \succ_{\rm xa} G(h)$ by Statewise Dominance.

So the Diamond Paradox only works if we exclude Bob's bitterness.

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1
Carol		

G	h	t
Ann	1	1
Bob	0	0
Carol	$-\gamma$	$-\gamma$

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

Or, suppose we include the decision-maker, Carol, as a third agent.

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1
Carol		

G	h	t
Ann	1	1
Bob	0	0
Carol	$-\gamma$	$-\gamma$

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

Or, suppose we include the decision-maker, Carol, as a third agent.

Carol may feel some *guilt* $-\gamma$ if she treats Bob unfairly.

F	h	t
Ann	1	0
Bob	0	1
Carol	0	0

G	h	t
Ann	1	1
Bob	0	0
Carol	$-\gamma$	$-\gamma$

It is important to stipulate that these payoff tables provide a complete description of the problem.
We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

Or, suppose we include the decision-maker, Carol, as a third agent.

Carol may feel some *guilt* $-\gamma$ if she treats Bob unfairly.

Then her payoffs would be as shown in the revised table.

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1
Carol	0	0

G	h	t
Ann	1	1
Bob	0	0
Carol	$-\gamma$	$-\gamma$

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

Or, suppose we include the decision-maker, Carol, as a third agent.

Carol may feel some *guilt* $-\gamma$ if she treats Bob unfairly.

Then her payoffs would be as shown in the revised table.

So if $\succeq_{\rm xp}$ is any impartial, Pareto *ex post* SWO, then $F(h) \succ_{\rm xp} G(h)$ and $F(t) \succ_{\rm xp} G(t)$. Hence $F(h) \succ_{\rm xa} G(h)$ by Statewise Dominance.

Complete specification in the Diamond Paradox

F	h	t
Ann	1	0
Bob	0	1
Carol	0	0

G	h	t
Ann	1	1
Bob	0	0
Carol	$-\gamma$	$-\gamma$

It is important to stipulate that these payoff tables provide a complete description of the problem.

We must exclude any *anticipation*, *bitterness*, or *guilt* from the story —or stipulate that these effects are *already accounted for* in the payoff tables.

Or, suppose we include the decision-maker, Carol, as a third agent.

Carol may feel some *guilt* $-\gamma$ if she treats Bob unfairly.

Then her payoffs would be as shown in the revised table.

So if $\succeq_{\text{xp}}$ is any impartial, Pareto *ex post* SWO, then $F(h) \succ_{\text{xp}} G(h)$ and $F(t) \succ_{\text{xp}} G(t)$. Hence $F(h) \succ_{\text{xa}} G(h)$ by Statewise Dominance.

So the Diamond Paradox only works if we exclude Carol's guilt from the model (Sen,1985).

Appendix D

Hybrids of *ex ante* egalitarianism

and

ex post egalitarianism

Ben-Porath, Gilboa & Schmeidler: min-of-means

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.

To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
Ann	1	1
Bob	0	0

MoM: 0.3

E	h	t
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
Ann	1	1
Bob	0	0

MoM: 0.3

E	h	t
Ann	$1/2$	$1/2$
Bob	$1/2$	$1/2$

MoM: 0.5

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
Ann	1	1
Bob	0	0

MoM: 0.3

E	h	t
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

Ben-Porath, Gilboa & Schmeidler: min-of-means

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
Ann	1	1
Bob	0	0

MoM: 0.3

E	h	t
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

The **A**-weighted mean of F is $0.3 \cdot 1 + 0.4 \cdot 0 + 0.2 \cdot 0 + 0.1 \cdot 1 = 0.4$.

Ben-Porath, Gilboa & Schmeidler: min-of-means

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \; \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \; \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
Ann	1	1
Bob	0	0

MoM: 0.3

E	h	t
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

The **A**-weighted mean of F is $0.3 \cdot 1 + 0.4 \cdot 0 + 0.2 \cdot 0 + 0.1 \cdot 1 = 0.4$.
The **B**-weighted mean of F is $0.4 \cdot 1 + 0.3 \cdot 0 + 0.1 \cdot 0 + 0.2 \cdot 1 = 0.6$.

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
Ann	1	1
Bob	0	0

MoM: 0.3

E	h	t
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

The **A**-weighted mean of F is $0.3 \cdot 1 + 0.4 \cdot 0 + 0.2 \cdot 0 + 0.1 \cdot 1 = 0.4$.
The **B**-weighted mean of F is $0.4 \cdot 1 + 0.3 \cdot 0 + 0.1 \cdot 0 + 0.2 \cdot 1 = 0.6$.
The **C**-weighted mean of F is $0.1 \cdot 1 + 0.2 \cdot 0 + 0.4 \cdot 0 + 0.3 \cdot 1 = 0.4$.

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
Ann	1	1
Bob	0	0

MoM: 0.3

E	h	t
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

The **A**-weighted mean of F is $0.3 \cdot 1 + 0.4 \cdot 0 + 0.2 \cdot 0 + 0.1 \cdot 1 = 0.4$.
The **B**-weighted mean of F is $0.4 \cdot 1 + 0.3 \cdot 0 + 0.1 \cdot 0 + 0.2 \cdot 1 = 0.6$.
The **C**-weighted mean of F is $0.1 \cdot 1 + 0.2 \cdot 0 + 0.4 \cdot 0 + 0.3 \cdot 1 = 0.4$.
The **D**-weighted mean of F is $0.2 \cdot 1 + 0.1 \cdot 0 + 0.3 \cdot 0 + 0.4 \cdot 1 = 0.6$.

Ben-Porath, Gilboa & Schmeidler: min-of-means

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
Ann	1	1
Bob	0	0

MoM: 0.3

E	h	t
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

The **A**-weighted mean of F is $0.3 \cdot 1 + 0.4 \cdot 0 + 0.2 \cdot 0 + 0.1 \cdot 1 = 0.4$.
The **B**-weighted mean of F is $0.4 \cdot 1 + 0.3 \cdot 0 + 0.1 \cdot 0 + 0.2 \cdot 1 = 0.6$.
The **C**-weighted mean of F is $0.1 \cdot 1 + 0.2 \cdot 0 + 0.4 \cdot 0 + 0.3 \cdot 1 = 0.4$.
The **D**-weighted mean of F is $0.2 \cdot 1 + 0.1 \cdot 0 + 0.3 \cdot 0 + 0.4 \cdot 1 = 0.6$.
Thus, the min-of-means for F is $\min\{0.4, 0.6\} = 0.4$.

Ben-Porath, Gilboa & Schmeidler: min-of-means

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	*h*	*t*
Ann	1	0
Bob	0	1

MoM: 0.4

G	*h*	*t*
Ann	1	1
Bob	0	0

MoM: 0.3

E	*h*	*t*
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

The **A**-weighted mean of G is $0.3 \cdot 1 + 0.4 \cdot 1 + 0.2 \cdot 0 + 0.1 \cdot 0 = 0.7$.

Ben-Porath, Gilboa & Schmeidler: min-of-means

Ben-Porath, Gilboa & Schmeidler (1997) axiomatically characterized the so-called *min of means* method of evaluating social prospects.
To illustrate this, consider the following four matrices of *weightings*:

$$\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}.$$

Consider the following three social prospects:

F	h	t
Ann	1	0
Bob	0	1

MoM: 0.4

G	h	t
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Bob	0	0

MoM: 0.3

E	h	t
Ann	$^1/_2$	$^1/_2$
Bob	$^1/_2$	$^1/_2$

MoM: 0.5

The **A**-weighted mean of G is $0.3 \cdot 1 + 0.4 \cdot 1 + 0.2 \cdot 0 + 0.1 \cdot 0 = 0.7$.
The **B**-weighted mean of G is $0.4 \cdot 1 + 0.3 \cdot 1 + 0.1 \cdot 0 + 0.2 \cdot 0 = 0.7$.

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The **C**-weighted mean of G is $0.1 \cdot 1 + 0.2 \cdot 1 + 0.4 \cdot 0 + 0.3 \cdot 0 = 0.3$.

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The **D**-weighted mean of G is $0.2 \cdot 1 + 0.1 \cdot 1 + 0.3 \cdot 0 + 0.4 \cdot 0 = 0.3$.
Thus, the min-of-means for G is $\min\{0.7, 0.3\} = 0.3$.

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Thus, MoM(F) >MoM(G), so F is ranked above G, in accord with our *ex ante* egalitarian intuitions.

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Thus, MoM(F) $>$MoM(G), so F is ranked above G, in accord with our *ex ante* egalitarian intuitions.
Finally, the min-of-means for E is $\frac{1}{2}(0.1 + 0.2 + 0.3 + 0.4) = 0.5$.

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MoM: 0.5

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Thus, MoM(E) >MoM(F), so E is ranked above F, in accord with our *ex post* egalitarian intuitions.

Gajdos & Maurin: weighted cross-iterative evaluations

Social prospect:

$$
\begin{array}{ccccc}
F = \begin{array}{c|c|c|c|c|} \cline{2-5} \mathrm{A} & & & & \\ \cline{2-5} \mathrm{B} & & & & \\ \cline{2-5} \end{array} & \xrightarrow{\epsilon} & \epsilon(F) = \begin{array}{|c|} \hline \ \\ \hline \ \\ \hline \end{array} & \xrightarrow{\omega} & \omega * \epsilon(F) \\
\downarrow \omega & & & & \downarrow \\
\omega(F) = \begin{array}{|c|c|c|c|} \hline \ & \ & \ & \ \\ \hline \end{array} & & & & \downarrow \\
\downarrow \epsilon & & & & \downarrow \\
\epsilon * \omega(F) & \longrightarrow & \phi(F) & = & \lambda\, \epsilon * \omega(F) + (1-\lambda)\, \omega * \epsilon(F)
\end{array}
$$

Let F be a social prospect (indexed by states and people).
Let ω be an *ex post* social welfare function, which acts on *ex post* social outcomes expressed as utility vectors.
Apply ω to each outcome of F, to get a vector $\omega(F)$ *ex post* social welfare levels, indexed by states.

Let ϵ be a sort of "generalized expectation" operator, which acts on *ex ante* prospects, expressed as state-contingent payoff vectors.
Apply ϵ to each individual prospect contained in F, to get a vector $\epsilon(F)$ of "expected utilities", indexed by individuals.

We then define $\omega * \epsilon(F) := \omega[\epsilon(F)]$ (a sort of generalized XAE evaluation).
Likewise, we define $\epsilon * \omega(F) := \epsilon(\omega[F])$ (a generalized XPE evaluation).

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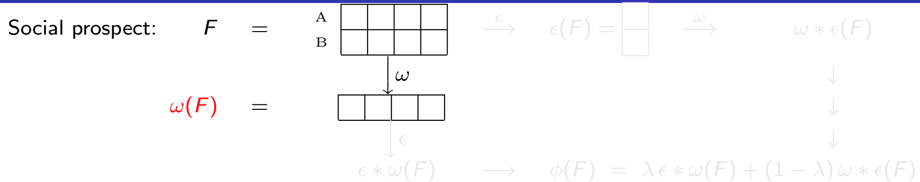

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Gajdos & Maurin: weighted cross-iterative evaluations

Social prospect:

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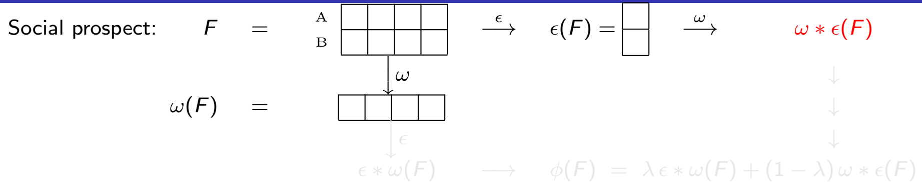

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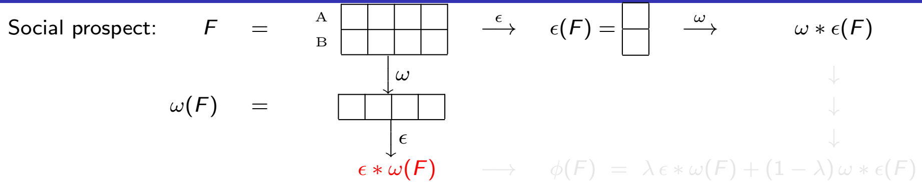
Gajdos & Maurin: weighted cross-iterative evaluations



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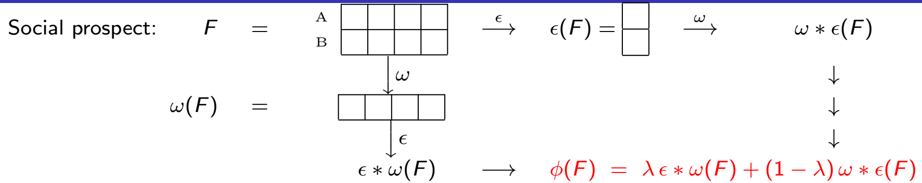

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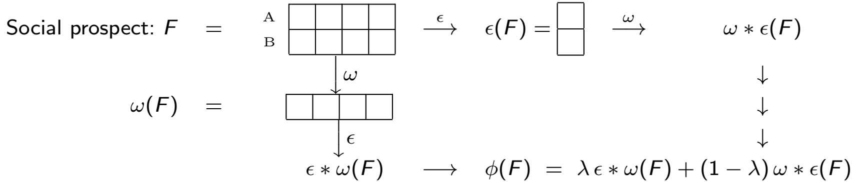

BG&S ('97) showed that, if ω and ϵ are M.o.M's, then ϕ is also an M.o.M.

Gajdos and Maurin (2004) investigated other *ex ante* social evaluations with a similar structure (where ϵ and ω are *not* necessarily M.o.M's).

They allowed λ to depend on F (subject to certain restrictions), and refered to ϕ as a *weighted cross-iterative* (WCI) evaluation.

In effect, ϕ is a sort of "compromise" between XAE and XPE.

G&M axiomatically characterized various other families of WCI evaluations.

For example, certain axioms single out social evaluations of the form

$$\phi(F) \quad := \quad \lambda \cdot \underline{M}(F) + (1-\lambda)\overline{M}(F), \qquad \text{where}$$

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Gajdos & Maurin: weighted cross-iterative evaluations

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Gajdos & Maurin: weighted cross-iterative evaluations

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\omega(F) = \begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} \arrow[d, "\epsilon"] & & \downarrow \arrow[d] \\
\epsilon * \omega(F) \arrow[r] & \phi(F) = \lambda(F)\,\epsilon * \omega(F) + (1 - \lambda(F))\,\omega * \epsilon(F) & \downarrow
\end{tikzcd}$$

BG&S ('97) showed that, if ω and ϵ are M.o.M's, then ϕ is also an M.o.M.
Gajdos and Maurin (2004) investigated other *ex ante* social evaluations with a similar structure (where ϵ and ω are *not* necessarily M.o.M's).

They allowed λ to depend on F (subject to certain restrictions), and refered to ϕ as a *weighted cross-iterative* (WCI) evaluation.

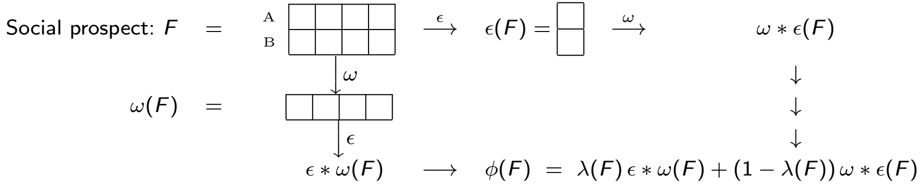
In effect, ϕ is a sort of "compromise" between XAE and XPE.

G&M axiomatically characterized various other families of WCI evaluations.

For example, certain axioms single out social evaluations of the form

$$\phi(F) \quad := \quad \lambda \cdot \underline{M}(F) + (1 - \lambda)\overline{M}(F), \qquad \text{where}$$

$\underline{M}(F) := \min\{\epsilon * \omega(F), \omega * \epsilon(F)\}$ and $\overline{M}(F) := \max\{\epsilon * \omega(F), \omega * \epsilon(F)\}$.



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